THE
PROCEEDINGS
OF
THE PHYSICAL SOCIETY

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THE PHYSICAL SOCIETY

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SIR ARTHUR EDDINGTON, F.R.S.

President 1930–32

THE EXPANDING UNIVERSE

Presidential Address by
SIR ARTHUR EDDINGTON, F.R.S.

PROCEEDINGS OF THE PHYSICAL SOCIETY

ERRATA

Two Precision Condenser Bridges, *by* A. Campbell, M.A.

Vol. 43, pp. 564–568 (1931)

Page 566, equation (6): *for* $M\lambda - m\mu$ *read* $(M\lambda - m\mu)$.

Page 566, equation (7): *for* $+ m(m - \mu)$ *read* $- m(m - \mu)$.

Page 566, equations (9) and (10):

$\quad$ *for* $m(m - \mu)/\rho$ *read* $m(m - \mu)\omega^2/\rho$.

Page 567, line 28: $\quad$ *for* $11000\,\Omega$ *read* $1000\,\Omega$.

Page 567, line 34: $\quad$ *for* $500\,\Omega$ *read* $2000\,\Omega$.

expand too much they burst. On this point at least I can speak reassuringly. Our bubble of a universe is not going to burst—for the best of reasons. It burst quite a long while ago.

§ 2. ASTRONOMICAL EVIDENCE

Let us first take an astronomical view of the subject. Up to the farthest limits surveyed by our telescopes space is dotted with numerous islands—the spiral nebulae. They are so far apart that light takes about a million years to cross from one island to the next. Each island turns out to be a galaxy of stars. Naturally our own island galaxy has a particular importance in our eyes; it is estimated to

contain from 5 to 50 thousand million stars. Ordinarily when we refer to the stars we mean the stars of our own island, for it is only recently that it has become possible to discern individual stars in some of the nearest of the other islands. We view our own system of stars from within; it has a flattened disc-like form, and the circuit of the Milky Way marks the plane in which it extends. The other island galaxies are seen from without, and it is easier to recognize at once their shape and character; they are usually found to be flattened and rotating like our own system.

The external galaxies are so remote that we could not expect to detect any apparent movement; but for some years we have been able in favourable cases to measure their radial velocities in the line of sight by the shift of their spectral lines. More recently the distances of some of them have been determined by a fairly reliable method. In two or three of the nearest spiral nebulae (including the great Andromeda nebula) E. Hubble was able to discover cepheid variables and to measure their magnitudes and periods. We know sufficient about the variables in our own system to understand that a cepheid variable of given period is a quite definite standard of light; we can use it in place of a standard candle. If you see a standard candle anywhere and note how bright it appears to you, you can easily calculate how far off it is; in just the same way the astronomer when he sees his "standard candle" in the midst of a nebula can calculate the distance of the nebula. That method unfortunately applies only to the nearest galaxies. For those farther off less satisfactory methods are used, and the distances assigned are to be described rather as estimates than as measures. Still they should serve well enough for statistical purposes.

When we survey the collected data as to distances and radial velocities we find an extraordinary state of affairs. The velocities are large—very much larger than ordinary stellar velocities. Also the more distant galaxies have the bigger velocities, and there is a fairly regular law of increase, the velocity being roughly proportional to the distance. The most striking thing of all is that the galaxies are with remarkable unanimity going away from us.

With regard to the last statement it is worth while examining the actual results in more detail. About 90 line-of-sight velocities of spiral nebulae have been measured, and amongst these only five are approaching us*. At first sight it may seem illegitimate to pass over the minority as insignificant. But we notice that all five exceptions are among the very nearest of the nebulae; and since we are dealing with an effect which increases with the distance, it is natural that we should have to go out some distance before it predominates over accidental irregularities (including observational error) and displays itself uniformly. The five approaching velocities are comparatively small and are at least partly attributable to our inappropriate standard of reference. The velocities as published are relative to the sun; but the sun is describing an orbit inside our galaxy with an orbital velocity of between 200 and 300 km./sec.; it is more significant to consider approach and recession with respect to our galaxy as a whole, so that a correction for the sun's orbital

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* The Andromeda nebula and two satellites (close to it in space and within range of its attraction) are counted as one nebula.

motion should be applied. It is found that this considerably reduces the approaching velocities. I think it will turn out that further small corrections are required, and that ultimately these five nebulae will be found to have small velocities of recession; for even one genuine exception would be very difficult to account for.

Let us exclude the nebulae which are more or less hesitating in our neighbourhood by drawing a sphere of rather more than a million light-years radius round our galaxy; we can then say definitely that, in the vast region beyond, the nebulae are unanimously running away from us. More than 80 have been observed to be moving outwards and not one has been found coming in to take their place. It is an obvious inference that in the course of time the region will be evacuated. The nebulae will all be out of reach of our telescopes, unless we increase our telescopic power to keep pace. I find that an observer of nebulae will have to double the aperture of his telescope every 1300 million years merely to keep up with their recession. Sir James Jeans delights in telling us that we have billions of years before us in which to find out all that can be found out about the universe. I suggest, however, that there is urgency as regards the spiral nebulae; if we leave it too late, there will be none left to examine.

I do not wish to insist on these astronomical facts dogmatically. No doubt there is a possibility of error and misinterpretation. But if we ask what is the picture of the universe now in the minds of those who have been engaged in practical exploration of its large-scale features—men not likely to be moved overmuch by ideas of curvature of space or the covariance of the Riemann-Christoffel tensor—the foregoing is their answer. Their picture is the picture of an *expanding universe*. We have dwelt on the fact that the nebulae are running away from ourselves; but the rule that at double the distance the velocity is doubled means that they are running away from each other as much as they are running away from us. This is the law according to which the points on a rubber balloon recede from one another when the balloon is being inflated; the mutual recession of two points is proportional to their distance apart. There is a uniform inflation of the system of the galaxies extending at least as far as our observations extend; and they extend a good distance. The latest recruit is a nebula in Leo distant more than 100 million light-years, which is receding with a velocity of 19,500 km./sec.; that is about the speed of an alpha particle.

§3. DE SITTER'S THEORY

In beginning with the astronomical observations I am deviating from the historical order, for it was the theory of relativity that first led us to look for a phenomenon of this kind. In 1917 W. de Sitter found that on one of two alternative hypotheses the light of remote objects would be displaced towards the red, and he suggested the motions of the spiral nebulae as a discriminating test. At that time only three velocities of spiral nebulae had been published, and these somewhat lamely supported his theory by a majority of 2 to 1. There the matter rested until 1923 when V. M. Slipher kindly supplied me with his (then unpublished) measures of the velocities of 40 spiral nebulae for use in my forthcoming book. As the

majority had now become 37 to 3, I was able in my *Mathematical Theory of Relativity* to present de Sitter's theory with much more favourable emphasis. It should be mentioned, however, that the effect now found is not strictly the original de Sitter effect which was proportional to the square of the distance. It was not realized at first that de Sitter's hypothesis required also a larger linear effect of the same kind; that was found when the theory had been developed in the clearer form associated especially with the name of G. Lemaitre (1927). When Hubble found a roughly linear relation between distances and velocities from his discussion of the observations in 1929 he was probably unaware that theory inclined to a linear rather than a quadratic relation.

§4. THE COSMICAL CONSTANT

We must go back a little before de Sitter. In 1915 Einstein had by his general theory of relativity brought the world to good order. The quantum, it is true, continued to pursue its communistic activities outside the reach of his law; but otherwise the state of things anywhere within finite distance of the observer might be considered fairly settled. Einstein thereupon became uneasy about infinity. I need not enter into the particular difficulties occasioned by having to employ boundary conditions at infinity; but Einstein saw that by far the simplest way of getting rid of these difficulties was to get rid of infinity. Instead of infinite space, take a "finite but unbounded" space—a spherical space of radius R. (We can always go back to infinite space, if it turns out that the facts of nature require it, by proceeding to the limit when R becomes indefinitely great.) Einstein effected this change by altering his law of gravitation in empty space from its original form $G_{\mu\nu} = 0$ to its more accurate form $G_{\mu\nu} = \lambda g_{\mu\nu}$, where λ is a very small coefficient called the *cosmical constant*. The added term is negligible in the ordinary applications of relativity to the solar system.

The cosmical constant λ is so fundamental in the theory of the expanding universe that we must spend a little time considering why it appears at all. The law $G_{\mu\nu} = \lambda g_{\mu\nu}$ is the expression of the fact that our measurements of length are not absolute, and that a length can only be measured relatively to another length treated as standard. If we translate it from symbols into words, the law states that the radius of spherical curvature of every three-dimensional section of the world is the same constant length—the same number of metres. Or we can put the statement the other way round—*what we call a metre at any place and in any direction is a particular constant fraction of the radius of curvature of the world for that place and direction*.

The law of gravitation is simply the ideal definition of the metre. It tells us how lengths at different places and times are to be compared. They are said to be equal if each bears the same ratio to the world-radius in its own neighbourhood. Clearly the first essential in our conception of space is that it should provide such a criterion of equality; without it space would be mere emptiness and not a metrical background. The curved space of relativity theory supplies its own standard units,

the various radii of curvature in different directions. We adopt these natural units in our measurements through the intermediary of the metre, for the metre is (if the above law of gravitation is true) a constant fraction of the natural unit.

We may perhaps ask, How is it that our practical standard, the metre bar which we move about in space and time, agrees so well with the ideal definition? Since the question relates to the behaviour of a material system of some 10^{27} particles distributing themselves according to quantum laws which are not yet fully formulated, we cannot give a very detailed answer. But what else could the metre bar do but reproduce the natural unit? It is selected for regularity of behaviour; its extension is determined by some constant set of equations. If we have the extension of our material bar on one side of the equations, what extension can we put on the other side? The only candidate for the position on the other side of the equation is the extension of the natural unit characteristic of the region where the bar is situated. Therefore we infer that the extension of the metre bar will be proportional to the extension of the natural unit contained in the space where it lies. That is what the law of gravitation asserts.

The ratio of the metre to the radius of curvature is determined by λ. If λ is zero the ratio is zero and the connection breaks down. We are left with a space which does not fulfil the first conditions of a medium of measurement; and the relativity theory is laid open to criticisms such as have been brought forward by Prof. Whitehead (mistakenly, I think, as regards the existing theory) as failing to provide a "basis of uniformity" for spatial measurement. For this reason the cosmical term $\lambda g_{\mu\nu}$ is essential in relativity theory. When it was first introduced it might have been regarded as a fancy addition, but it is now seen to be indispensable*. I see that Einstein has recently proposed to take λ equal to o; that seems to me an incredibly retrograde proposal.

It is a curious fact that whilst the rather difficult conception that length is relative to the motion of a reference body is now a commonplace principle of physics, there is very little recognition of the much more elementary relativity of length that I have been referring to, viz. that we can only recognize the ratio of two lengths. If all lengths were altered in the same ratio the change would be undetectable and meaningless. When we speak of a linear constant of nature such as the radius of the unexcited hydrogen atom, what do we mean by saying that it is constant? We can only mean that its ratio to some other length is constant. Practically we compare it with a metre bar and say that it is a constant fraction of the international metre; but this, though true, is clearly not fundamental. It holds because the metre bar is governed by an elaboration of the same laws which govern the hydrogen atom, and both regulate their extension by reference to the same standard. The point I am here stressing is that our geometry—or description of space—must describe a standard which is everywhere available for comparison when we want to state the dimensions of a physical system. A geometry which does not supply any linear standard is useless for physical purposes. The second point is that

* Cf. H. Weyl, *Raum-Zeit-Materie*, p. 297 (English edition). "The cosmological factor (λ) which Einstein added to his theory later is part of ours from the very beginning."

although our practical measurements are made, not by direct comparison with the simple standards contained in the geometry of space but by comparison with complicated material systems, nevertheless the result is the same as if we had used the ideal standards. The interchangeability of the ideal standards and the practical reckoning is expressed symbolically by the law $G_{\mu\nu} = \lambda g_{\mu\nu}$*.

§5. THEORY OF THE EXPANDING UNIVERSE

The immediate result of this change in the law of gravitation was the appearance (in theory) of two universes—the Einstein universe and the de Sitter universe. Both were possible theoretically; both involved spherical space; but since de Sitter's universe required an apparent recession of distant objects whilst Einstein's did not, it was hoped that observation of the spiral nebulae would discriminate between them. They were called static universes, for unlike our "expanding universe" they remained unchanged for any length of time. But it was realized later that the changelessness of de Sitter's universe was due to the fact that he had left it entirely empty, so that there was nothing in it that *could* change†. Einstein's universe was therefore the only form of a material universe that could remain motionless. The situation has been summed up by saying that Einstein's universe contained matter but no motion and de Sitter's contained motion but no matter.

The actual universe with both matter and motion cannot correspond exactly to either of these two simple models. The only question is, Which is the best choice for a first approximation? Shall we put a little motion into Einstein's world of matter, or a little matter into de Sitter's world of motion which is waiting to move something? The question is not so urgent now, for we are no longer restricted to the two extremes. We have now the whole chain of intermediate solutions of $G_{\mu\nu} = \lambda g_{\mu\nu}$, from which we can pick the one which has the right proportion of matter and motion to correspond with what we know of our own universe. These solutions seem to have been first given by A. Friedman in 1922. They were re-discovered by Lemaître in 1927, who developed the astronomical consequences elegantly and exhaustively. His work seems to have remained unknown until last year when he called my attention to it in connection with problems then engaging attention‡. In the meantime the solutions had been again discovered by H. P. Robertson.

The intermediate solutions are expanding universes. At one end we have Einstein's universe with no motion and therefore in equilibrium; then as we proceed along the series we get more and more rapid expansion until we reach de Sitter's universe which forms the limit. It is the limit because whilst the

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* This interpretation of the law of gravitation was given by the lecturer in 1921. For fuller details, see *Mathematical Theory of Relativity*, §§ 65, 66 and *The Nature of the Physical World*, chap. 7.

† The ideal changelessness of de Sitter's universe failed as soon as anything, e.g. a spiral nebula, was put into it.

‡ The original paper (which is rather inaccessible) has now been reprinted in *Monthly Notices R.A.S.* **91**, 483 (1931).

expansion has been increasing the density has been decreasing all the way along the series, so that by the time de Sitter's form is reached we are left with a rapid expansion but nothing to expand.

The meaning of this series can be better understood by beginning at the de Sitter end. Just before we remove the last vestiges of matter so as to obtain the empty de Sitter world, the expansive tendency has free play with nothing to counteract it. When matter is inserted gravitation tends to hold the mass together and so opposes the expansion. The more matter we put in, the more the expansion is counteracted. For a particular density the gravitational attraction of the matter will just balance the expansive force, and we have complete equilibrium. This corresponds to the Einstein universe. If we take still higher density gravitation will prevail over expansive force and we have a contracting universe.

A point which I think should be particularly stressed is that Einstein's universe is unstable*. It corresponds to an exact balance between gravitation and the expansive force. Now suppose there happens to be a very small expansion. The gravitation between the galaxies is weakened by the increase of distance so that it no longer counterbalances the expansive force; thus further expansion ensues. Similarly a small contraction will lead to further contraction. The slightest disturbance will therefore cause the delicately balanced Einstein world to topple into a state of continually increasing expansion or continually increasing contraction.

It will be seen that our expectation of finding an effect like that apparently manifested in the motions of the spiral nebulae has been greatly strengthened since it was first suggested as a hypothesis. Firstly the term $\lambda g_{\mu\nu}$ which gives rise to it is now understood to be an integral and necessary part of the law of gravitation and not a fancy addition. Secondly, instead of the effect appearing in one of two alternative theories, we now find that the only form of universe which does not give rise to the effect is unstable, so that even if the universe originally had this form it would not retain it.

§6. OBSERVED AND PREDICTED RECESSION

Attempts have been made to settle whether expansion rather than contraction would be expected theoretically, but I am not sure that they have been successful. At any rate they involve additional hypotheses and do not depend simply on the law of gravitation. Apart from this ambiguity of sign relativity theory definitely predicts the type of phenomenon observed in the motions of the spiral nebulae. But the prediction is qualitative and not quantitative; no hint is given as to the scale of the phenomenon. Theoretically we expect to find a systematic recession of sufficiently remote objects, but we have no idea as to whether "sufficiently remote" means at nebular distances or at distances 10^6 or 10^{60} times greater. It is a stroke of luck that (if the observations are to be trusted) the required distance is well within range of our telescopes. The theory gives the magnitude of the recession in

* A. S. Eddington, *Monthly Notices*, **90**, 668 (1930). This paper gives an account of Lemaître's theory, including the more elementary mathematical part.

terms of the cosmical constant λ, but hitherto we have had no knowledge of the magnitude of λ except such as may be inferred by fitting the theory to the observations.

Are we on safe ground in identifying the observed recession of the spiral nebulae with the expansion effect predicted by theory? The weak point is the absence of any prediction of the magnitude; and the question of magnitude happens to be of considerable importance. It is one thing to say that our universe cannot remain the same size for ever and that if we wait long enough it will expand indefinitely; it is another thing to say that the universe is blowing up so fast that it has doubled its diameter within geological times. Theory is responsible only for the first statement; when the spiral nebulae, not content with confirming it, proceed to add the barely credible second statement, their value as witnesses becomes suspect. For this reason some astrophysicists have not unnaturally inclined to the view that the whole notion of recession of the spiral nebulae is a misinterpretation of the red-shift of their light; they do not necessarily doubt the relativity prediction of an expanding universe, but they point out that it may well be a very much slower change altogether undetectable by astronomical observation.

A theory put forward by Zwicky as to the cause of the red-shift of the nebular light has attracted some attention. He suggests that it is not a Doppler effect but is due to cumulative loss of energy of the quantum of light on its way to us by the gravitational perturbations which it exerts on particles of matter. We cannot very well rule out this hypothesis as impossible unless we claim to know all about light quanta—a claim which I certainly shall not make. But I think the present position of Zwicky's hypothesis is not generally realized. In his original paper he supported it by a mathematical investigation intended to show that the hypothesis led to results of the right order of magnitude. There was a mathematical mistake in the paper which invalidated the result, and I understand that this investigation has been withdrawn. Dr Zwicky continues to advocate his suggestion in a more indefinite form, as he is quite entitled to do; but naturally we regard differently a suggestion which is now wholly speculative from one which appeared to be supported to some extent by a confirmatory investigation.

§7. THEORETICAL VALUE OF THE COSMICAL CONSTANT

I have recently obtained a result which I think clears up the situation. I have found a theoretical value of the cosmical constant from a study of the wave equation of the electron*. Using this value we can make a purely theoretical calculation of the speed of recession of remote objects; the result is found to agree with the observed motions of the spiral nebulae. Thus there seems to be no doubt that the observed motions are genuine and are due to the cosmical expansion.

For the last three years I have been much occupied with a theory of some of the natural constants which appear in physics, particularly the constant $hc/2\pi e^2$, or 137. A year ago† it seemed likely that the investigation would develop so as to

embrace two other constants, viz. the mass-ratio of the electron and proton, and the cosmical constant λ. I may say at once that I have got no further with the theory of the mass-ratio of the proton and electron*; but if I am not mistaken the theory of the cosmical constant has proved unexpectedly simple. It does not involve directly my theory of the other constants. Nor does it involve the auxiliary mathematical developments connected with my theory of 137. But I am not sure that it can be grasped without an understanding of the general conceptions that I have been applying—or misapplying—in that theory.

The theory of the cosmical constant is in fact more directly connected with the explanation of the law of gravitation to which I have already referred. We have seen that our standard metre adjusts itself as a constant fraction of the radius of curvature of space-time, so that it is an intermediary by which we compare the dimensions of a physical system with the radius of the world. Although the metre rod is a useful practical intermediary, it is a red herring in theoretical investigations; for these will naturally have a simpler and more illuminating form when the radius of the world is introduced directly.

I suppose that nowadays we should look on the wave equation for the hydrogen atom as an example of the most fundamental type of physical equation. It is of simple form because it deals with the most elementary interplay of entities. It determines the linear spread of the charge—or the probability, or whatever the stuff may be—that surrounds the nucleus. From what has already been said the spread can only be expressed as a ratio, the ratio of distances in the atom to some other standard of distance. The equation determines the ratio, and therefore both participants in the ratio must figure in the equation. The standard, as we have seen, is the radius of world curvature; and the wave equation must say just as much about the radius of the world as it says about the spread of the constituents of the atom. We have not one equation saying how the spread of the hydrogen atom is fixed and another equation saying how the curvature of the world is fixed; we have an equation saying how the spread of the hydrogen atom is fixed in relation to the curvature of the world.

The equation can be adulterated by making substitutions based on other physical equations, which will perhaps not distort its simplicity of form though they distort its simplicity of meaning. In that way we obtain the wave equation as ordinarily written, which drags in the material standard metre and gives the extension of the hydrogen atom as compared with it—a comparison which is doubtless of greater practical utility than a comparison with the world radius. Here I want to see as far as possible into the inner significance of the wave equation, and I must take the equation unadulterated.

I have said that the wave equation says as much about the world-radius R as it does about the hydrogen atom. Now the ordinarily accepted form of the wave equation never mentions R; in fact it ostensibly refers to entirely flat space-time. The ordinary wave equation agrees with experiment, so we may not reject or

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* Since this was written advance has been made, and a theoretical value 1847·60 for the mass-ratio seems to follow naturally from the present theory of the cosmical constant.

substantially amend it. The two requirements are reconciled if we realize that the ordinary wave equation does indeed contain a term involving R but *it writes it in a disguised form*. There is only one possible term which could contain R in disguise; it is the term commonly attributed to the proper mass of the electron. I think we might have expected that; the well-known relation between mass and space-time curvature in relativity theory suggests that the world radius might disguise itself in a mass term.

We ordinarily regard the wave equation as describing the probability of various distances r within the atom; but it is equally an equation for the probability of various radii R of the world, since it is only the ratio of r to R that counts. The wave function ψ refers to the probability of various sets of simultaneous values of these two variables; that is to say it is a function $\psi(r, R)$. We know the term in r, for that is not disguised in the ordinary equation; it is proportional to $1/r$, and since the whole equation may be divided through by an arbitrary factor we shall suppose it so written that this term is definitely $1/r$. The mass term then becomes mc^2/e^2; but that, as I have explained, is its disguised form. Can we say precisely what is its undisguised form?

Since the equation determines only the ratio r/R, homogeneity demands that R shall occur in the form $1/R$ to correspond with $1/r$. There may be a numerical coefficient, but we shall leave this aside for the moment.

We thus reach the conclusion that the wave equation is built up round a kind of "core" consisting of two terms

$$\frac{1}{r}, \quad \frac{1}{R}.$$

We can see moreover that this corresponds precisely to the geometry of the problem. When we say that an electron is in a spherical space of radius R and at a distance r from a proton, we are effectively stating bipolar coordinates. It is distant r from one fixed point, the proton; it is distant R from another fixed point, the centre of the spherical world. The fact that one of these distances is outside ordinary physical space-time is not relevant here; that will be cared for by the way in which the remaining terms of the equation, which introduce the space-time coordinates, are linked on to the core. But we have to take note of one kind of asymmetry between our two bipolar coordinates. The datum is that there are N electrons (all the electrons of the universe) at a distance R from the centre of the world, and of these just one is at a distance r from the proton. Obviously it would be a very different problem if there were N electrons at a distance r from the proton, of which just one was in spherical space. Thus r and R are not simply interchangeable, and the number N has to be introduced.

Omitting detailed discussion it appears that this difference causes the "core" terms to be

$$\frac{1}{r}, \quad \frac{\sqrt{N}}{R}.$$

If you wonder at the square root I may remind you that in wave mechanics, if ψ is the wave function normalized so as to represent one electron, $\psi\sqrt{N}$ is the wave

function normalized to represent N electrons. I must also add that R refers not to the present radius of space but to the radius of the universe in a stationary state, i.e. arranged as an Einstein world; for the wave equation gives the stationary states (*eigen* states) of a system, and this applies just as much to the universe of N electrons as to the one electron in the atom.

We have thus penetrated the disguise of the term mc^2/e^2 and revealed it to be $\sqrt{N}/R$. The equation

$$\frac{mc^2}{e^2} = \frac{\sqrt{N}}{R}$$

is my new result. If you have not been able to follow the steps of the deduction, you will at least see that no adjustable factors have been introduced to bring about agreement with observation artificially.

Numerical results

The ordinary relativity theory of the Einstein world had already provided another relation between N and R*, so that with the new result we are able to determine N and R separately and hence the cosmical constant λ which is equal to $1/R^2$. The following are the values obtained:

Equilibrium radius of universe $(R) = 1\cdot010 \cdot 10^{27}$ cm. $= 1068$ million light-years
$= 328$ megaparsecs.

Mean density in equilibrium state $= 1\cdot05 \cdot 10^{-27}$ gm. cm.$^{-3}$.

Total mass of universe $= 2\cdot143 \cdot 10^{55}$ gm. $= 1\cdot08 \cdot 10^{22} \times$ mass of sun.

Number of electrons in universe $(N) = 1\cdot29 \cdot 10^{79}$.

Cosmical constant (λ) $= 9\cdot8 \cdot 10^{-55}$ cm.$^{-2}$.

Thus far the results are such as could not well be falsified by observation. But having determined λ and R we can at once find the limiting speed of recession of the spiral nebulae which by Lemaître's theory is $c/R\sqrt{3}$ per unit distance; this gives

528 kilometres per second per megaparsec.

This is the full recession undiminished by countervailing gravitational contraction. The correction for gravitation between the nebulae can be calculated only from estimates of the average density of matter in space; it is unlikely that it exceeds 50 km./sec. and it may well be trifling. We therefore predict a motion rather less than, but not much less than, the figure above stated.

This is in excellent agreement with the observed value which has generally been stated in round numbers as 500 km./sec. per megaparsec.

Incidentally it is a rather gratifying confirmation of our astronomical scale of distances. When we reflect on the number of intermediate stages by which we step from the standard metre to distances of a hundred million light-years, and on the possibility of unforeseen errors at each stage, it is no small achievement to find the scale confirmed to within 10 per cent.

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* The total mass M of the universe is approximately NM, where M is the mass of a proton. The Einstein relation is $M = \frac{1}{2}\pi R$ in gravitational units or $GM/c^2 = \frac{1}{2}\pi R$ in c.g.s. units.

§8. THE TIME-SCALE

If the expanding universe is accepted as an established fact, its most immediate reaction is on the time-scale of evolution. Three main time-scales have been favoured at one time or another which we may distinguish as "short," "intermediate" and "long." No one now has a good word for the short Kelvin time-scale; and practically our choice lies between the intermediate scale giving the sun an age of the order 10^{10} years and the long scale giving an age of $5 \cdot 10^{12}$ years. When there is no definite evidence one way or the other the longer time-scale naturally gets the preference. The more time allowed, the more can happen; so the policy of the evolutionist is to grab as much time as possible. This, rather than any striking success, accounts for the popularity of the long time-scale in recent years.

The hypothesis of the long time-scale came about through Einstein's theory which gave the total amount of energy in a given mass of matter. We knew just how much energy there was in the sun and could calculate how long it would maintain the radiation if it could all be released. To release it all it is necessary that protons and electrons should annihilate one another, thereby undoing the lock which fastens the energy in. This idea of the source of a star's energy seems to have been first mentioned by me in 1917[*]. It was the only adequate source that could be suggested at the time; but in 1920 a possible alternative was recognized in the energy released by the transmutation of hydrogen into higher elements. This alternative, however, suffices only for the intermediate time-scale. I do not think anything very decisive has been found for or against either theory (annihilation of protons and electrons, or transmutation of hydrogen) or either time-scale (long or intermediate). When it was necessary to choose one or the other, I have, like other time-grabbers, generally preferred the long scale. Last year, however, in considering the dynamics of our own rotating galaxy, I was much impressed by the strong argument that it furnished for the intermediate scale[†].

With the universe doubling its radius every 1300 million years it is obvious that the long time-scale of billions of years is altogether incongruous. It is true that we cannot set any definite limit to the time occupied by the first slow development of the expansion. But if there were billions of years to choose from it is strange that the evolution of our own solar system should coincide with the relatively short interval between the "bursting of the bubble" and the complete dispersal of the galaxies.

§9. NATURE OF THE EXPANSION

For convenience we deal with a model spherical world, but it is not necessary to assume that the actual universe is at all closely spherical. If it started from an equilibrium configuration (and it is difficult to imagine any other kind of beginning which would not be unaesthetically abrupt) it must then have been spherical; but it may have developed lop-sidedly, and it may now be any shape, indeed space

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[*] *Monthly Notices*, **77**, 611.

[†] *The Rotation of the Galaxy; Halley Lecture* (Oxford Univ. Press).

need not even be closed in any legitimate sense of the word. Such irregularity would not affect our calculation of the recession of the spiral nebulae, which represents a uniform expansive tendency present in every region of space; the form of the universe as a whole only affects the calculation of the countervailing gravitational attraction which is believed to be relatively small.

The expansion is relative to our ordinary standards, e.g. the standard metre or the wave-length of cadmium light, which in turn depend on the scale of atomic phenomena. Moreover it does not affect anything but the intergalactic distances. Notwithstanding the expansive tendency the atoms, the earth, the solar system, the galaxy itself, all remain constant. The reason for this paradox can be seen as follows. Suppose that the sun and planets were all given enormous electric charges of the same sign; that would introduce a strong "expansive tendency" into the solar system. But the solar system would not become an expanding system; there would be an initial readjustment, but afterwards the planets would describe periodic orbits as before under the modified field of force. It is only if the charge on the planets were made so strong as to overbalance gravitation that the planets would abandon the periodic type of orbit and recede continually. There is thus a sharp demarcation between systems which exhibit the expansion of the universe and systems which do not—corresponding to the distinction between periodic and aperiodic phenomena. If the expansion had been operative in all types of phenomena equally we should have been entirely unable to detect it.

You may perhaps have noticed an apparent contradiction. The radius of the world is increasing relatively to the metre; on the other hand we have especially emphasized the fundamental principle that the metre is defined as a definite fraction of a world-radius. It is my fault for not distinguishing more precisely between the various radii of curvature. Space-time can have an appalling variety of twistiness, and I could not specify the particular curvature referred to without becoming unduly technical. Also the curvature in the empty space between material objects, referred to in the definition of the metre, is not the same as the average curvature of a region including material objects.

Probably the radius of the universe is already increasing at a rate exceeding the velocity of light, and the rate is continually increasing. Mention of velocities greater than the velocity of light always arouses suspicion; but if any careful statement of the assertions of relativity theory on this point is consulted, it will be found that all is well.

§ 10. SEEING ROUND THE WORLD

In Einstein's spherical universe light can go round and round the world. In de Sitter's universe it cannot go round, because the journey would require an infinite time. Our expanding universe is intermediate, and we ask how light will behave in it.

In the sequence of intermediate stages between the Einstein and de Sitter forms there is a definite stage at which circumambulation ceases to be possible. Our universe is now well past this point, so that light can no longer go round the

world in finite time. The circumference of the world is expanding, and light is like a runner on an expanding track with the winning-post receding faster than he can run.

We assume that the universe was originally in the Einstein state of equilibrium. In those early days light and other radiation went round and round the world until it was absorbed. (It is worth mentioning that, in making one circuit, radiation would on the average meet with obstruction equivalent to 31 cm. of water, so that cosmic rays of moderate penetrating power could make several circuits. The average obstruction has now become less.) This merry-go-round lasted during the very early stages of expansion. But when the world had expanded to 1·003 times its original radius the bell rang for the last lap; light-waves then running will make just one more circuit before $t = \infty$; those which started later will never get round.

Somewhat later, when the increase of radius had become 1·073*, the last half-lap was announced. From that moment onwards it has become impossible for ligh to travel half-way round; so that corresponding to any star there is a region of the universe which its present radiation will never reach. And if light cannot reach no other causal influence can reach, for no signal can travel faster than light. That was the moment when *the bubble burst*. The star and its antipodal region are as disconnected from one another as the fragments of a bubble; their only causal connection is through the past time before the breakage.

After what I have just been saying, it may seem paradoxical to add that in the expanding universe we are (theoretically) able to see round the world—the process vividly, if inaccurately, described as "observing the back of one's head." It is true that the line of vision passes through the antipodal region, which has now become impassable; but the waves now reaching our eyes are those which passed through the region a long while ago, before the bubble broke.

The longer the light travels the more it becomes reddened. The reddening of the light from any object follows a simple rule; the wave-length is increased in the ratio of the radius of the world at the time of observation to the radius when the light was emitted. We have seen that light which has been round the world must have started before the expansion reached 1·003, so that at the time of its emission the radius was practically the initial Einstein radius. Unfortunately we can make only the roughest guess at the present value of the expansion; but I should be surprised if it is less than 4 : 1, and will take that figure as an illustration. In that case the light from the "back of one's head" will have its wave-length increased four-fold; it will accordingly be infra-red.

This result has an interesting bearing on the problem of the cosmic penetrating radiation. According to the statements of the experimenters the rays are coming more or less equally from all directions; this points to a source symmetrical about the earth. Now astronomers know of no celestial source with the least approach to directional symmetry about the earth, except the whole universe. In any case it

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* The critical values 1·003 and 1·073 were first given by de Sitter. He pointed out a mistake in my own calculation just in time for me to give the correct values in *Monthly Notices*, **90**, 673 (1930). The exigencies of page-proofs prevented my acknowledging their source.

would seem that if cosmic rays are being generated in one system or one galaxy they should be generated in all, so that cosmic rays must have been poured out all over the universe from the beginning of evolution. We have seen that in general they will not be absorbed until they have been many times round; so I imagine that what we are now observing is the long accumulation of past ages. This seems to be the only way in which the popular theory, if it is to be accepted at all, can be put into satisfactory shape. But if so, no trust can be placed in the present attempts to determine the origin of the rays from calculations of their wave-length. The observed wave-length is much greater than the original wave-length which alone provides a clue to their origin. Presumably most of the rays were emitted during the long period when the world was still near the Einstein condition, so that a correcting factor probably greater than 4 is needed. Perhaps when accurate determinations of the wave-lengths are known and likely sources are decided on, the factor can be fixed definitely; that is to say, we may from observing the cosmic rays—which preserve remembrance of an earlier state of the world—determine how much our universe has expanded from its original condition. That would be a notable contribution to astronomy.

§ 11. CONCLUSION

All change is relative; and what we have called the theory of the "expanding universe" might also be called the theory of the "shrinking atom." It was for that reason that it was not inappropriate to investigate the problem by examining the wave equation of the atom.

"Les hommes, les animaux, les pierres grandissent en s'approchant et deviennent énormes quand ils sont sur moi. Moi non. Je demeure toujours aussi grand partout où je suis*."

I am always the same; it is the universe that is swelling. In this problem there is, I think, more justification than usual for our egocentric outlook; for I may remind you that the cosmos not only swells but it bursts, and I do not see how we can regard the bursting as the reflection of some reciprocal process undergone by ourselves. But it is a good scientific exercise to consider another point of view, though I must warn you that it has no philosophical moral in this instance. So for a last glance at the problem let us take the view of a cosmic being, whose body is composed of intergalactic spaces and swells as they swell; or rather we must now say, it keeps the same size, for he will not admit that it is he who has changed. Watching us for some few thousand million years he sees us gradually shrinking; atoms, animals, planets, even the galaxies, all share the same contraction; only the intergalactic spaces remain the same. The earth spirals round the sun in an ever-decreasing orbit. Naturally he will not accept our year as a unit of time; it is the period of a continually shrinking orbit. Presumably he will relate his units of time and length so that the velocity of light is constant. Our years will then decrease in geometrical progression in the scale of cosmic time. Owing to the property of geometrical

* Anatole France. The speaker is the dog Riquet.

progressions an infinite number of years will add up to a finite cosmical time; so that our $t = \infty$ is an ordinary finite date in the cosmic calendar. On this date stars, planets, atoms are doomed to disappear; for when $t = \infty$ the universe has expanded to infinite radius in our reckoning, and we have shrunk to zero in the reckoning of the cosmic being.

We walk the stage of life, performers of a drama for the benefit of the cosmic spectator. As the scenes proceed he notices that the actors are growing smaller and the action growing quicker. When the last act opens the curtain rises on midget actors rushing through their parts at frantic speed. Smaller and smaller. Faster and faster. One last microscopic blurr of intense agitation. And then nothing.

DIFFUSION FOR THE INFINITE PLANE SHEET

By A. T. McKAY, M.Sc.,

British Boot, Shoe and Allied Trades Research Association

ABSTRACT. The paper deals with a problem of diffusion of moisture into an infinite plane sheet of hygroscopic material, when subject to two different surface conditions. The method of treatment follows along the lines of a previous paper*. Further, a method is developed whereby the diffusivity and surface constants can be evaluated from experimental data. In order to facilitate the practical application of the methods propounded to this and similar diffusion problems, tables have been specially calculated giving the first four roots of each of the four equations $\left.\begin{matrix}\tan\\\cot\end{matrix}\right\} x = \pm\, x \cdot \left.\begin{matrix}\tan\\\cot\end{matrix}\right\} \lambda$. The particular design of these tables has been chosen to allow of ready interpolation.

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§1. INTRODUCTION

THE present paper is intended as an extension of one previously published in the *Proceedings of the Physical Society*†. The problem considered is the diffusion of moisture into an infinite plane sheet of hygroscopic material, when subject to two different surface conditions.

Apart from any physical application which the derived formulae may have it is hoped that the methods are worthy of attention. The use of operational methods as adopted in this paper commends itself for several reasons, namely that (i) the necessity of effecting Fourier expansions is avoided; (ii) the final result is derived in the form of a complex integral from which moments and approximations can readily be found; and (iii) the procedure is both direct and brief.

A further purpose of the work is to draw attention to the use of moments for the evaluation of the constants of an equation. While the principle of least squares is of undoubted merit when it can be applied, it very frequently happens to be quite unmanageable and the method of moments, so ably exploited by statisticians, must be resorted to. In the present instance, were data known *a priori* to follow the derived law it would appear a very lengthy and difficult process to evaluate the constants by methods that differ from those propounded.

The evaluation of a criterion of the curve is a small but useful innovation which proves of great service in the first survey of experimental data.

In order to render this paper of practical value the roots of the four transcendental equations occurring in the theory of diffusion have been computed. The form of these tables has been chosen to allow of interpolation, a most desirable quality in practical analysis.

* *Proc. Phys. Soc.* **42**, 235 (1930).

† *Ibid.*

§2. STATEMENT OF PROBLEM AND DEFINITION OF SYMBOLS

An infinite plane sheet of hygroscopic material, initially moisture-free, has one face maintained at saturation and the other face in the presence of a fixed humidity. It will be supposed that the absorption process is expressed by the usual partial differential diffusion equation together with a surface resistance, and that the concentration within the material is a linear function of the equivalent relative humidity. It is required to find the expression for the quantity of moisture absorbed in a given time interval and to develop a method of determining the diffusivity and surface constants from observational data.

The symbols used in this paper are the same as those in the before-mentioned paper with the following additions and modifications:

The origin of coordinates is taken at the plane in the vicinity of the arbitrary humidity.

a is the thickness of the sheet;
θ_0 the concentration *outside* the plane $x = 0$;
θ_a the concentration *at* the plane $x = a$;
$Q_0 = a\theta_0$;
$Q_a = a\theta_a$;
$(Q_a + Q_0)/Q_a = r$.

§3. SOLUTION OF THE DIFFERENTIAL EQUATION

We require to solve the differential equation

$$\frac{\partial^2 \theta}{\partial x^2} = \frac{1}{k} \frac{\partial \theta}{\partial t} \qquad \ldots\ldots(1),$$

subject to the conditions:

$$
\begin{aligned}
&(a) \ \theta = \theta_a && \text{when} \quad x = a \\
&(b) \ \partial\theta/\partial x = -c\,(\theta_0 - \theta)/a && \text{when} \quad x = 0 \\
&(c) \ \theta = 0 && \text{when} \quad t = 0
\end{aligned}
\qquad \ldots\ldots(2).
$$

Since the method of solution is parallel to that already given in the previous paper it is only necessary to give the results of the analysis. The operational solution of the problem is therefore included in the following two expressions:

$$\theta = \frac{c\,(\theta_a - \theta_0 \cosh \sigma a)\sinh \sigma x + (\sigma a\,\theta_a + c\theta_0 \sinh \sigma a)\cosh \sigma x}{(\sigma a \cosh \sigma a + c \sinh \sigma a)} \qquad \ldots\ldots(3),$$

$$Q = \frac{\sigma a\,Q_a + c\,(Q_a + Q_0)\tanh(\sigma a/2)}{\sigma a\,(\sigma a \coth \sigma a + c)} \qquad \ldots\ldots(4).$$

Before proceeding to the general interpretation of these last two equations it will be useful to obtain several special cases.

(i) *The concentration value at the steady state.*

We require the value of θ as $t \to \infty$. Now it is known that σ can be regarded as of the order $t^{-\frac{1}{2}}$ so all that is necessary for our purpose is to find the limit of the right-hand side of equation (3) as $\sigma \to 0$. This limit is found to be:

$$\theta_m = \{c\,(\theta_a - \theta_0)\, x + (\theta_a + c\theta_0)\, a\}/a\,(1 + c) \qquad \ldots\ldots(5).$$

(ii) *The maximum amount absorbed.*

This is given by direct integration of equation (5), whence:

$$Q_m = \{Q_a + (Q_a + Q_0)\, c/2\}/(1 + c) \qquad \ldots\ldots(6)*.$$

(iii) *Approximation when t is small.*

When t is small, σ can be regarded as of large order; hence from equation (4),

$$Q = (Q_a + Q_0)/\sigma a - Q_0/(\sigma a + c) \qquad \ldots\ldots(7).$$

The first term is readily interpreted but the most useful interpretation of the second term depends on the order of $\sigma a/c$. Further information about the expansion of this term will be found in a paper by Sumpner†.

Returning now to equation (4) we may express Q as a contour integral by means of the Bromwich rule and we arrive at the result:

$$Q = \frac{Q_a}{2\pi i} \int_L e^{ktz \cdot a^2} f(z) \cdot \frac{dz}{z} + \frac{c\,(Q_a + Q_0)}{2\pi i} \int_L e^{ktz/a^2} f(z) \cdot \frac{\tanh \frac{1}{2}z^{\frac{1}{2}}}{z^{\frac{1}{2}}} \cdot \frac{dz}{z} \quad \ldots\ldots(8),$$

where

$$f(z) = (z^{\frac{1}{2}} \coth z^{\frac{1}{2}} + c)^{-1}.$$

The integrand of the right-hand side of equation (8) is a single-valued function of z with poles at $z = 0$, $z = -\beta_n^2$ where $n = 1, 2, \ldots$ and β_n is the nth positive root of the transcendental equation $\qquad\qquad \beta_n, n$

$$y \cot y + c = 0 \qquad \ldots\ldots(9).$$

The poles of $\tanh z^{\frac{1}{2}}$ are zeros of $f(z)$ and therefore make no contribution to the integral. It will be noted that the pole $z = 0$ merely contributes the value of Q_m which we have already found. We only then require to consider the contributions from the poles $z = -\beta_n^2$.

Now,

$$\operatorname*{Lim}_{z \to -\beta_n^2} f(z) \cdot (z + \beta_n^2) = 2\beta_n^2/(c + c^2 + \beta_n^2),$$

whence

$$Q = Q_a \left\{ \frac{1}{1+c} - \sum_1^\infty \frac{2e^{-k\beta_n^2 t/a^2}}{(c + c^2 + \beta_n^2)} \right\} + c\,\frac{(Q_a + Q_0)}{2} \left(\frac{1}{1+c} - \sum_1^\infty \frac{2e^{-k\beta_n^2 t/a^2} \tan \frac{1}{2}\beta_n/\frac{1}{2}\beta_n}{(c + c^2 + \beta_n^2)} \right)$$
$$\ldots\ldots(10).$$

* A point of interest perhaps worth noticing is the following: If a diffusion process somewhat like that herein contemplated plays a part in the phenomena of "sorption" we see from equation (6) that by attributing to c, the surface constant, a directional property an explanation could be offered for the hysteresis effect known to be manifested by many materials in the course of absorption and desorption of water vapour.

† *Proc. Phys. Soc.* **41**, 241 (1929).

The equation (10) is the solution sought and gives Q explicitly in terms of t. We note that when the surface constant is either infinite or zero Q can be expressed in terms of the simple diffusion function defined in the previous paper.

§4. DETERMINATION OF MOMENTS

μ_n The curve represented by equation (10), together with its asymptote and the quantity-axis, bounds a finite area. Let μ_n be such that

$$\mu_n Q_m = \int_0^\infty t^n \, (Q_m - Q) \cdot dt.$$

Now from equation (8) we have

$$Q = \frac{Q_a}{2\pi i} \int_M e^{ktz/a^2} \, F(z) \cdot \frac{dz}{z},$$

where

$$F(z) = \frac{1 + (cr \tanh \tfrac{1}{2} z^{\frac{1}{2}})/z^{\frac{1}{2}}}{c + z^{\frac{1}{2}} \coth z^{\frac{1}{2}}} ;$$

therefore $\quad \displaystyle\int_0^\infty t^n \, (Q_m - Q) \, dt = - \frac{Q_a}{2\pi i} \int_0^\infty \int_N e^{ktz/a^2} \, t^n F(z) \cdot \frac{dz}{z} \cdot dt,$

N where N is a contour that just avoids the origin.
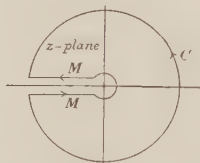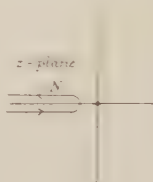
Fig. 1.   Fig. 2.

Since the real part of kz/a^2 is negative for every point of N,

$$\int_0^\infty e^{ktz/a^2} t^n \, dt = \left(\frac{-a^2}{kz} \right)^{n+1} n!,$$

so that $\quad \displaystyle\int_0^\infty (Q_m - Q) \, t^n \, dt = - \frac{Q_a}{2\pi i} \, n! \int_N \frac{F(z)}{z} \left(\frac{-a^2}{kz} \right)^{n+1} dz \qquad \ldots\ldots(11).$

Since the integrand of this expression is regular on and outside C and the integral round the infinite circle vanishes, we have that the integral along the path M is zero. Whence the integral along path N is equal to minus the integral round the origin. We deduce then that

$$\left.\begin{aligned} Q_m &= Q_a F(0) \\ \mu_0 Q_m &= - (a^2/k) \, Q_a F'(0) \\ \mu_1 Q_m &= + \tfrac{1}{2} (a^4/k^2) \, Q_a F''(0) \end{aligned}\right\} \qquad \ldots\ldots(12).$$

Hence $\quad\quad \mu_0 = \dfrac{a^2 \, (rc^2 + 5rc + 8)}{12k \, (1 + c) \, (2 + rc)} \qquad\qquad \ldots\ldots(13),$

$$\mu_1 = \frac{a^4}{360k^2} \, \frac{(3rc^3 + 24rc^2 + 61rc + 16c + 96)}{(1 + c)^2 \, (2 + rc)} \qquad \ldots\ldots(14).$$

§ 5. EQUATION CONSTANTS AND CURVE CRITERION

We define the curve criterion R thus:

$$R = \mu_1/\mu_0{}^2 \qquad \dots\dots(15).$$

Whence from equations (13) and (14),

$$R = \frac{2}{5}\frac{(2+rc)(3rc^3 + 24rc^2 + 61rc + 16c + 96)}{(rc^2 + 5rc + 8)^2} \qquad \dots\dots(16).$$

The maximum and minimum values of R are given when

$$3(r^3 - r^2)c^4 - (r^3 + r^2)c^3 - (39r^2 - 24r)c^2 - (57r^2 - 48r)c - (88r - 128) = 0$$
$$\dots\dots(17),$$

and since c must be positive we are only concerned with the positive roots. When $r > \frac{16}{11}$ there is only one positive root which will give a minimum value of R. The effective maximum will be $1\cdot2$ corresponding with $c = 0$. When $r < \frac{16}{11}$ there are two positive roots of equation (17) which provide the appropriate range for R. The experimental conditions being fixed, i.e. r being known, the range for R can thus be determined. If observational values yield a value of R within the pre-determined range, two positive values of c can be derived from equation (16). The method of continued approximation or Newton's method of sequences will prove most expedient for the latter purpose. It will be noted that when $R = 1\cdot2$, $c = 0$ or ∞, and consideration of the physical interpretation of this shows that the ambiguity is irremovable.

We shall now discuss two special cases, namely those in which

(*a*) the concentration outside the face $x = 0$ is saturation, i.e. $r = 2$; and

(*b*) the concentration outside the face $x = 0$ is zero, i.e. $r = 1$.

Case (a). From equations (16) and (17) we find

$$R = \tfrac{6}{5}\{1 - c/(4 + c)^2\} \qquad \dots\dots(18),$$

and the range

$$1\cdot125 \leqslant R \leqslant 1\cdot2 \qquad \dots\dots(19).$$

The values of c and k yielded by equations (13), (14) and (18) are

$$c = \left\{\frac{24}{3 \pm 5\sqrt{(4\cdot8R - 5\cdot4)}} - 4\right\} \qquad \dots\dots(20),$$

$$k = 2a^2/15\mu_0\{1 \pm \sqrt{(4\cdot8R - 5\cdot4)}\} \qquad \dots\dots(21).$$

Case (b). In this case we find:

$$R = \frac{6}{5}\frac{(3c^2 + 15c + 18)(3c^2 + 15c + 32)}{(3c^2 + 15c + 24)^2} \qquad \dots\dots(22),$$

and the range for R is

$$1\cdot2 \leqslant R \leqslant 1\cdot225 \qquad \dots\dots(23).$$

Writing $V = 3c^2 + 15c + 24$ we reduce equation (22) to a quadratic in V. R being known it is therefore a simple matter to find c. The corresponding value of k is then obtained by use of equation (13).

§6. EXAMINATION OF EXPERIMENTAL DATA

As is evident from the definition in § 4, $\mu_0 . Q_m$ and $\mu_1 . Q_m$ are the area and first moment respectively of the region enclosed by the diffusion curve, the asymptote and the line $t = 0$. The care to be exercised and the methods to be adopted in the derivation of Q_m, μ_0 and μ_1 from experimental data have been dealt with at length in the previous paper and need not therefore be repeated. Since the range for R is determinate when the experimental conditions are fixed the first enquiry is as to whether the ratio $\mu_1/\mu_0{}^2$ falls within this range. When this requirement is satisfied equation (16) must be solved by numerical methods to give the two values of c. From equation (13) two corresponding values of k are derived. It does not appear possible to remove the ambiguity and there is nothing to be done but to make tests of fit with the alternative pairs of values of the constants.

§7. ACKNOWLEDGMENT

Table 1: The first four roots of the equation $\tan x = -x \tan \lambda$
for values of λ from 0° to 90° at 5° intervals.

$\lambda°$	1st root	2nd root	3rd root	4th root
0	3·1416	6·2832	9·4248	12·5664
5	2·8936	5·8127	8·7703	11·7665
10	2·6976	5·5120	8·4453	11·4553
15	2·5434	5·3238	8·2775	11·3142
20	2·4196	5·1986	8·1781	11·2354
25	2·3176	5·1098	8·1124	11·1850
30	2·2310	5·0432	8·0655	11·1497
35	2·1559	4·9911	8·0300	11·1233
40	2·0892	4·9487	8·0018	11·1025
45	2·0288	4·9132	7·9787	11·0855
50	1·9729	4·8826	7·9590	11·0712
55	1·9204	4·8556	7·9419	11·0588
60	1·8702	4·8313	7·9267	11·0478
65	1·8214	4·8090	7·9128	11·0378
70	1·7732	4·7883	7·9000	11·0286
75	1·7249	4·7686	7·8879	11·0199
80	1·6756	4·7496	7·8764	11·0115
85	1·6246	4·7309	7·8651	11·0034
90	1·5708	4·7124	7·8540	10·9956

Table 2: The first four roots of the equation tan $x = x \tan \lambda$
for values of λ from 0° to 90° at 5° intervals.

$\lambda°$	1st root	2nd root	3rd root	4th root
0	—	3·1416	6·2832	9·4248
5	—	3·4344	6·8213	10·1510
10	—	3·7225	7·1859	10·5004
15	—	3·9562	7·3862	10·6588
20	—	4·1248	7·5030	10·7453
25	—	4·2446	7·5783	10·7996
30	—	4·3320	7·6308	10·8371
35	—	4·3985	7·6699	10·8649
40	—	4·4508	7·7004	10·8866
45	0·0000	4·4934	7·7252	10·9041
50	0·6837	4·5292	7·7461	10·9189
55	0·9205	4·5600	7·7641	10·9316
60	1·0798	4·5872	7·7800	10·9429
65	1·2002	4·6116	7·7943	10·9531
70	1·2973	4·6340	7·8075	10·9624
75	1·3789	4·6549	7·8198	10·9712
80	1·4498	4·6748	7·8315	10·9796
85	1·5131	4·6939	7·8427	10·9876
90	1·5708	4·7124	7·8540	10·9956

Table 3: The first four roots of the equation cot $x = x \tan \lambda$
for values of λ from 0° to 90° at 5° intervals.

$\lambda°$	1st root	2nd root	3rd root	4th root
0	1·5708	4·7124	7·8540	10·9956
5	1·4451	4·3488	7·2865	10·2639
10	1·3390	4·0879	6·9665	9·9432
15	1·2481	3·9044	6·7860	9·7890
20	1·1686	3·7712	6·6737	9·7008
25	1·0977	3·6704	6·5975	9·6436
30	1·0330	3·5910	6·5420	9·6033
35	0·9728	3·5264	6·4995	9·5729
40	0·9157	3·4722	6·4655	9·5490
45	0·8603	3·4256	6·4373	9·5294
50	0·8057	3·3846	6·4133	9·5127
55	0·7506	3·3478	6·3923	9·4983
60	0·6939	3·3141	6·3735	9·4856
65	0·6341	3·2827	6·3564	9·4740
70	0·5690	3·2530	6·3405	9·4633
75	0·4956	3·2245	6·3255	9·4532
80	0·4079	3·1968	6·3111	9·4435
85	0·2915	3·1692	6·2970	9·4341
90	0·0000	3·1416	6·2832	9·4248

A. T. McKay

Table 4: The first four roots of the equation $\cot x = - x \tan \lambda$
for values of λ from $0°$ to $90°$ at $5°$ intervals.

$\lambda°$	1st root	2nd root	3rd root	4th root
0	1·5708	4·7124	7·8540	10·9956
5	1·7202	5·1346	8·4930	11·7968
10	1·8930	5·4807	8·8551	12·1290
15	2·0791	5·7038	9·0330	12·2711
20	2·2589	5·8437	9·1325	12·3474
25	2·4156	5·9365	9·1957	12·3950
30	2·5438	6·0022	9·2395	12·4279
35	2·6468	6·0514	9·2720	12·4522
40	2·7300	6·0899	9·2973	12·4711
45	2·7984	6·1213	9·3179	12·4864
50	2·8558	6·1475	9·3351	12·4993
55	2·9051	6·1702	9·3500	12·5104
60	2·9482	6·1902	9·3632	12·5203
65	2·9867	6·2082	9·3751	12·5292
70	3·0217	6·2248	9·3860	12·5374
75	3·0541	6·2403	9·3962	12·5451
80	3·0845	6·2549	9·4060	12·5524
85	3·1135	6·2692	9·4155	12·5595
90	3·1416	6·2832	9·4248	12·5664

YOUNG'S MODULUS FOR TWO DIRECTIONS IN A STEEL BAR

By G. A. WEDGWOOD, M.Sc., East London College

ABSTRACT. The investigation described is a continuation of previous work* by the author on the extensions of thick cylindrical shells under internal pressure. Experiments were carried out to test whether the elastic constant E of the various steels from which the hollow cylinders were made is the same in two directions at right angles, one direction being along the axis of the original bar and the other across a diameter. The results show that for the steels dealt with Young's modulus is the same in the two directions referred to.

§ 1. INTRODUCTION

IN connection with some previous experimental work on the extensions of cylindrical shells under internal pressure the question arose as to the elastic isotropy of the steel bars from which the cylinders were made, and it was thought that further work on this point was desirable. The question is a reasonable one, as it might be expected that on account of the rolling the elastic properties of a round bar are not the same across a diameter as along the axis of the bar.

In the experiments about to be described an attempt was made to test if the value of Young's modulus is the same in the directions already referred to for the materials from which the cylinders of the previous investigation were made. The work involved the measurement of the longitudinal strains of specimens of necessity very short, their length being less than $1\frac{1}{2}$ in., the diameter of the original material; and this necessitated the use of a very small extensometer with which to make the measurements. The mirror extensometer with scale and telescope, as developed by Prof. E. H. Lamb† and now in frequent use, should be admirably suitable for the measurement of these very small strains; and an instrument of this kind which could deal with the extension of a 0·625 in. length bar of diameter about 0·25 in. was designed by the author, and made in the instrument shop of East London College. The accuracy of a determination of Young's modulus by means of the instrument should be well within $\frac{1}{2}$ per cent.; and for this reason it was thought desirable that the testing-machine employed should be capable of measuring loads within the same degree of accuracy.

§ 2. THE TENSILE TESTING-MACHINE

A small 1-ton testing-machine was available, and this, after some important modifications, was employed throughout the work. The machine was of the single-lever type‡ in which variable weights can be suspended from a knife-edge at a fixed

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* *Proc. Phys. Soc.* **41**, 366 (1929). † *Engineering,* **119**, 207 (1925).
‡ *Mechanical Testing,* **1**, 45 (1922).

distance from the fulcrum. The specimen was screwed into two rods, one of which was secured to the frame of the machine by a nut, and the other to the shorter arm of the lever, also by a nut. Between the lever and the upper of these two rods two degrees of freedom of movement were provided, and the alignment of the load applied to the specimen was made to coincide with the axis of the specimen. The leverage of the testing-machine was carefully determined by suspending a known
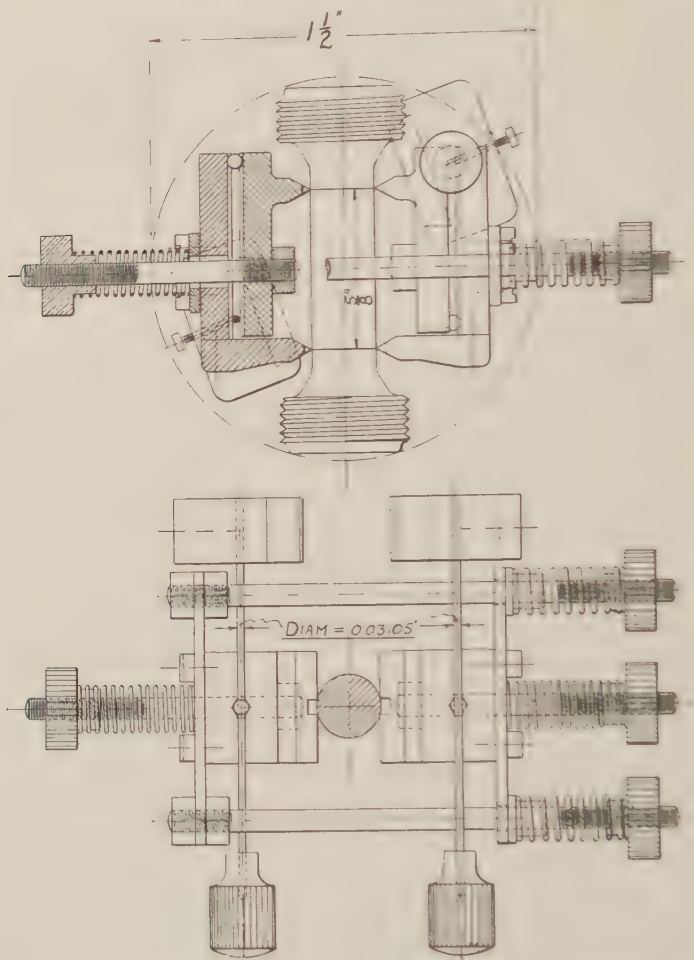
weight from the shorter arm and balancing this by weights suspended from the knife-edge at the other end ; and in this way, within 0·2 per cent., the leverage was found to be 22·3. In order to reduce the jarring action set up in placing a weight on the holder suspended from the end of the longer arm, a spring was inserted between the holder and the knife-edge; and in order to avoid inaccuracies in the extension measurements due to changes of temperature in the room, and also those due to vibration effects, the whole apparatus was installed in a cellar free from sunlight and draughts, and well away from disturbances set up by machinery in the College workshop.

§ 3. THE EXTENSOMETER

The mirror extensometer is shown mounted on the specimen in the two views in the figure, from which will be seen the slight modifications in its design to meet the requirements of the present work. The mean diameter of the rollers was about $\frac{1}{32}$ in. Each mirror-holder was balanced rotationally on its roller. The knife-edges of the instrument were notched; this ensured that the instrument could be attached to the specimen correctly with respect to the axis.

One of the chief difficulties encountered in the use of an extensometer of this kind is the making certain that there is no slipping at the points of contact between the knife-edges and the surface of the specimen. To get over this difficulty without clamping the instrument too tightly to the specimen, and also to be certain as to the exact distance between the knife-edges, two fine grooves about 0·002 in. deep and 0·625 in. apart were cut into the surface of the specimen; and particular care was taken when the instrument was being set up to see that the points of contact were in these grooves.

§ 4. THE SPECIMENS

By experimenting on a pair of specimens each of the same dimensions and made from the same bar of material, one of the specimens having its axis co-axial with the bar, the other having its axis along a diameter of the bar, it should be possible to test whether Young's modulus for the material was the same in these two directions. The specimens were on this account made in pairs from bars of diameter 1·5 in., the over-all length of each specimen being slightly less than this. Gas screw-threads of 0·125 in. were cut on the ends of each specimen; and between the ends the shank was turned down to a diameter of 0·2492 in., the corners being suitably filleted as shown in the figure.

Four pairs of specimens were made from the four bars the analyses of which are given in the previous paper*. It will be convenient here to describe these bars as: no. 1, 0·09 C; no. 2, 0·28 C; no. 3, 0·43 C; no. 4, $3\frac{1}{2}$ Ni. These eight specimens were normalized. Three other pairs of specimens were also made, one pair from a bar of ordinary commercial mild steel, and the other two pairs from two bars of wrought-iron, known as Toga iron; and these six specimens were not subjected to any heat treatment. Then in order to test the effect of the length of the specimen on the value of Young's modulus, four specimens 3 in. long, screwed at the ends and turned down to a diameter of 0·2495 in., were made from the bars nos. 1, 2, 3, and 4; and these were also normalized.

§ 5. THE TENSION EXPERIMENTS

If x is the extensometer scale-reading in centimetres as read by means of the spider-line of the telescope, d the mean diameter of the extensometer rollers in inches; L the distance from the scale to the mirror facing it, and l the distance between the extensometer mirrors, both L and l being in centimetres, the extension of the specimen is given by the formula $xd/(4L + 2l)$. In all the experiments L was

x

d

L, l

xd

* *Proc. Phys. Soc.* **41**, 366 (1929).

264·2 cm., and *l* was 2 cm. The load was applied in equal increments of 111·5 lb.; and by taking a set of eight or ten readings the extension produced by this increment of load in terms of *x* cm. of the scale-reading was easily determined. And since *d* is 0·03105 in., the distance between the knife-edges of the extensometer 0·625 in., and the diameter of the specimen 0·2492 in., Young's modulus for the fourteen short specimens can be conveniently calculated by means of the formula

$$E = 4·881 \times 10^7 \div x.$$

It was the usual practice to load the specimens several times before taking a set of readings. It was part of the procedure of the investigation to make four or five separate determinations of Young's modulus for each specimen; and in each of these experiments the instrument was removed from the specimen and reset in a different position. Provided the extensometer was carefully set up, it was an easy matter to make various separate determinations of Young's modulus each within 0·5 per cent. of the average of the values thus found. In all, some eighty separate determinations were made in the course of the present investigation, and in none of the experiments did the temperature in the vicinity of the specimen rise or fall by more than 0·05° C. The influence of temperature on the results is therefore practically of no account.

The readings taken in measuring the extensions of the three specimens made from bar no. 2, 0·28 C steel, are given in the table, in which is shown also the method

Table: Extensions of specimens from bar no. 2, 0·28 C

Specimen 1: Length 1½ in., co-axial with bar			Specimen 2: Length 1½ in., across a diameter of bar			Specimen 3: Length 3 in., co-axial with bar		
Load (lb.)	Scale-reading	Scale-difference	Load (lb.)	Scale-reading	Scale-difference	Load (lb.)	Scale-reading	Scale-difference
	No. cm.	(cm.)		No. cm.	(cm.)		No. cm.	(cm.)
—	1 19·43	(1) − (6) = 8·03	—	1 21·02	(1) − (6) = 7·97	—	1 19·52	(1) − (6) = 8·01
111·5	2 18·81	(2) − (7) = 8·01	111·5	2 19·43	(2) − (7) = 7·99	111·5	2 17·92	(2) − (7) = 8·01
223·0	3 16·21	(3) − (8) = 8·01	223·0	3 17·84	(3) − (8) = 7·98	223·0	3 16·33	(3) − (8) = 8·02
334·5	4 14·61	(4) − (9) = 8·01	334·5	4 16·24	(4) − (9) = 7·97	334·5	4 14·73	(4) − (9) = 8·02
446·0	5 13·00	(5) − (10) = 8·00	446·0	5 14·65	(5) − (10) = 7·97	446·0	5 13·12	(5) − (10) = 8·01
557·5	6 11·40	Mean = 8·012	557·5	6 13·05	Mean = 7·976	557·5	6 11·51	Mean = 8·01
669·0	7 9·80	Scale-difference for	669·0	7 11·44	Scale-difference for	669·0	7 9·91	Scale-difference for
780·5	8 8·20	load-increment of	780·5	8 9·86	load-increment of	780·5	8 8·31	load-increment of
892·0	9 6·60	111·5 lb. = 1·602 cm.	892·0	9 8·27	111·5 lb. = 1·595 cm.	892·0	9 6·71	111·5 lb. = 1·603 cm.
1003·5	10 5·00	$E = 30·5 \times 10^6$	1003·5	10 6·68	$E = 30·6 \times 10^6$	1003·5	10 5·11	$E = 30·4 \times 10^6$

of arriving at the scale-difference for the given load-increment 111·5 lb. The readings here recorded are representative of those obtained from experiments on the remaining fifteen specimens.

The following is a summary of the determinations of Young's modulus, the results being given in lb. × 10⁶ in.² for the fourteen short specimens, first for a direction co-axial with the original bar, and second for a direction at right angles to this axis: no. 1, 0·09 C, 30·4 and 30·5; no. 2, 0·28 C, 30·5 and 30·6; no. 3, 0·43 C, 30·5 and 30·5; no. 4, 3½ Ni, 29·8 and 29·8; mild steel, 30·6 and 30·5; wrought-iron, first bar, 29·7 and 29·5 to 28·7, second bar 29·7 and 28·5 to 28·1. The corresponding results for the four longer specimens are: no. 1, 0·09 C, 30·4; no. 2, 0·28 C, 30·4; no. 3, 0·43 C, 30·5; no. 4, 3½ Ni, 29·7.

§ 6. CONCLUSION

Within the degree of accuracy of the present work the results of the investigation show that the value of the elastic constant is the same at right angles to the axis of a steel bar of diameter 1·5 in. as along the axis of the bar. In the case of a wrought-iron bar of diameter 1·5 in. it was found that the value of Young's modulus for the specimens across a diameter of the bar was not constant, and was less than the value obtained for the specimens co-axial with the bar; this is to be expected on account of the direction of the slag impurities in wrought-iron. Comparing the results of the experiments carried out on the short steel specimens with the results of the work done on the corresponding longer specimens, it is seen that the values of the elastic constant are slightly less; this, of course, is also to be expected on account of end effects. It is interesting to note that the values of Young's modulus for the bars nos. 1, 2, 3, and 4 agree with the corresponding values recorded in the author's paper dealing with the extensions of cylindrical shells under internal pressure, and verifies the accuracy of the experiments carried out by means of a larger tensile testing-machine on much larger specimens. It would also appear that in the author's previous paper his speculation as to the non-isotropic nature of the material as an explanation of certain discrepancies in the results is not justified by the present piece of work.

§ 7. ACKNOWLEDGMENTS

In conclusion the author thanks the Council of East London College and Prof. E. H. Lamb for the facilities which have been placed at his disposal in connection with the investigation here described.

DISCUSSION

Dr J. S. G. Thomas referred to the fact that Young's modulus for a bar of wrought iron is less in the transverse than in the axial direction. The author attributed the difference to impurities in the iron. Presumably the slag inclusions would, as a result of hammering, arrange themselves parallel to the axis.

Mr T. Smith asked whether the same extensometer was used for long as for short specimens, so that the effective lengths of all specimens were equal.

Mr A. G. Warren: With such short specimens it is somewhat surprising that the author was able to obtain an accurate value of Young's modulus. There is a tendency in such cases for the skin to stretch more than the core. This can be shown for thin specimens by a comparison of reflection and transmission spectra.

Mr J. H. Awbery: Mr Wedgwood's result is certainly not quite what would have been anticipated, and it is therefore all the more interesting that he should have given us the facts.

Doubtless one could find out from his earlier paper, but perhaps he will allow me to ask whether the thick cylinders used in the earlier work were simply bored out, or whether any further process was applied to them?

AUTHOR's reply: The same extensometer was employed in all the experiments described in the paper. The accuracy of the determination of Young's modulus was found to be largely dependent on the use of two finely cut scratches on the surface of the specimen; these served the purpose of registering the two pairs of knife-edges of the instrument at a distance of 0·625 in. apart on the bar. I was surprised to find so little difference between the value of Young's modulus for the shorter specimens and the longer ones; I expected that in the case of the shorter specimens there would be a tendency for the skin to stretch more than the core in the vicinity of the fillets.

In reply to Mr AWBERY: The thick cylinders of the earlier work were very carefully prepared, and were not subjected to any treatment in their preparation other than the ordinary machine operations of turning, boring, and rymering.

A REMOTE ELECTRICALLY-RECORDING ACCELERO-METER WITH PARTICULAR REFERENCE TO WHEEL-IMPACT MEASUREMENTS

By F. AUGHTIE, Ph.D., M.Sc.,

of the Engineering Department, National Physical Laboratory

ABSTRACT. The instrument was developed for the measurement of the acceleration of the rear axle of a vehicle. It was necessary for the recording to be done remotely in order accurately to phase three records, two of acceleration and one of spring load. A new method of remote recording has been developed for the purpose.

The paper describes the design of the instrument, its principle of operation, and also, though briefly, the auxiliary equipment. The undamped natural frequency is 300 ∼ and the maximum reading is 25 *g*. Damping is provided by filling the casing with light mineral oil and, although less than critical, is sufficient to eliminate from the records any disturbance due to self-oscillation.

§ 1. INTRODUCTION

THE increasing use of the roads for transport purposes, together with the increased loads carried by vehicles, has brought into importance the question of road-wear together with the allied problems of noise, vibration and deterioration of the vehicle. It is clear that the form and details of the vehicle suspension—springs and tyres—play an important part in the destructive action of traffic on the road, and in an endeavour to obtain definite data an extensive research has been initiated by the Ministry of Transport, the work being carried out at the National Physical Laboratory. There being no suitable instruments available for the work at the time of its commencement, attention has, up to the present, been confined to the development of such instruments, and in particular of instruments for the measurement of the wheel-impact force. The present paper describes the design and principle of operation of an accelerometer developed for the work.

§ 2. THEORETICAL CONSIDERATIONS

The measurement of the force occurring between the wheel of a vehicle and the road during the impact caused by an irregularity in the road surface may be made either from the road or from the vehicle. For the former method a section of the road can be mounted upon a force-measuring device, usually a spring of some form, arrangements being made to record either the peak value of the force or its variation with time. While this method is attractive in that the measurement is made directly, it has several serious drawbacks. The need for mounting a section

of the road upon a sprung platform precludes the use of actually existing road surfaces, and causes a discontinuity in the road surface immediately in front of the section under examination. Moreover the magnitude of the force between the wheel and road-surface depends upon the deformation of the tyre and road-material: hence, unless the stiffness of the experimental road section (as suspended) is equal to that of the actual road, the method will obviously give erroneous results. But even if this condition can be fulfilled, there is a further error arising from the fact that the mass and stiffness of the experimental road-section will be concentrated in the platform and supporting spring, while in the actual road they are distributed through a considerable volume of material and this renders it impossible to obtain a platform equivalent to the road for impacts of differing wave-form.

Thus it is clear that while measurement from the road is comparatively easy, experimentally, there are serious theoretical objections to its use.

Except in the case of sprung-rim wheels, which are little used in this country, the load imposed upon the road by the wheel is the (algebraic) sum of the force applied to the axle through the springs and that necessary to accelerate the axle and fittings. The force required to accelerate the axle can be calculated from the mass constants of the axle and its linear and angular accelerations. The acceleration of a single wheel is insufficient for the determination of the impact force because, in general, the two wheels are not mutual centres of percussion and oscillation. That is, if one wheel is running on a smooth level surface while the second encounters an obstacle, the force between the first wheel and the road will be affected by the movement of the second wheel. The linear and angular accelerations of the axle are most conveniently determined by measuring the linear acceleration of two points on the axle. For an accurate determination of the total load at one wheel it is further necessary to know the force imposed by both springs individually, but when the distance between the centre of the wheel and the point of application of the spring load is small compared with the centre-distance of the wheels, a single load is sufficient for a reasonably accurate determination of the total force.

The problem thus reduces to that of measuring a single force and two linear accelerations*, and since all three (more particularly the latter two) vary rapidly and irregularly with time, it is necessary to record their variation in such a way that the three records can be accurately phased. This essential, namely the ability to phase the records accurately, makes it desirable if not imperative to record them upon a common chart and this in turn renders it necessary to record remotely at least two and preferably all three quantities.

Apart from the difficulty of constructing a recording device to operate successfully under the conditions of vibration existing on the back axle of a vehicle, it was impossible to rely on rods or other mechanical devices for the transmission of the reading from the measuring instrument to the recording unit, on account of errors introduced by backlash, inertia and elastic strain of the parts, particularly

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* It would be preferable to measure both spring loads, i.e. four quantities in all. Unfortunately standard recording apparatus has provision for only three and hence the scheme excludes the spring load on the opposite side to the wheel considered.

in view of the high magnification which will later be seen to be necessary. The transmission between the instrument and recorder-unit had therefore to be either hydraulic or electrical.

A hydraulic transmission offered attractions, but consideration showed that it could not be freed from risk of error due to thermal expansion, leakage and viscosity, and it was therefore abandoned in favour of an electrical transmission.

§ 3. PRINCIPLES OF THE METHOD

After consideration of the existing methods of remotely recording small displacements by electrical means, and an experimental trial of the only one which seemed likely to meet the conditions, they were rejected in favour of a method developed at the National Physical Laboratory for the purpose under consideration.

The fundamental principle of the method is illustrated in figure 1. Two coils *P* and *S* are mounted upon a laminated iron core, the magnetic circuit being completed by a laminated iron armature and a small air-gap whose length is determined by the quantity to be recorded. One of the coils *P* is supplied with alternating current of constant frequency and amplitude; the magnetic flux and hence
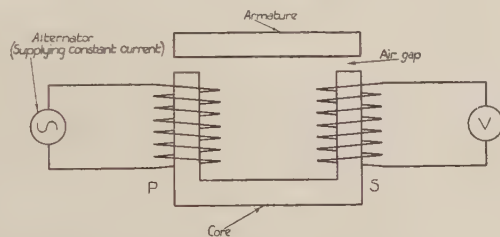
Fig. 1. Principle of operation of remote electrically-recording accelerometer.

the e.m.f. induced in the second coil *S* therefore depend upon the length of the air-gap, and a voltmeter connected to *S* will, if fitted with a suitably calibrated scale, indicate the magnitude of the air-gap and hence of the quantity to be measured. The voltmeter employed takes the form of a copper-oxide rectifier connected to an oscillograph which operates as a d.c. instrument, i.e. it does not follow the supply frequency which can therefore be raised to a value sufficiently high to give the required detail in the record. (The supply-frequency must, of course, not be so high as to render the rectifiers inoperative.)

It was considered preferable to use a constant primary current and measure the voltage (through the transformer), rather than to use a constant primary voltage and measure the current, because a power-supply of the frequency required was most conveniently obtained from a thermionic valve generator, and owing to the high internal resistance of this form of supply a constant current was obtained more readily than a constant voltage under conditions of variable load.

Trials with an experimental apparatus showed that the sensitivity of the arrangement fell as the frequency was increased owing, doubtless, to the reduced equivalent permeability of the iron core, and after consideration the supply-frequency was chosen as 1000 ~, this being about six times the anticipated natural frequency of

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the accelerometer. Above that frequency accurate records would not be possible, for mechanical reasons.

The experimental apparatus showed also that there was a considerable gain in sensitivity when the rectifiers were operated with an increased current-output and consequently with a lower internal resistance, the zero of the oscillograph being correspondingly suppressed, preferably by electrical means.

It was further found that owing to variation of the primary self-inductance, the supply current varied perceptibly as the air-gap varied, even with a reasonable swamping resistance in circuit, and this change was such as to reduce the overall sensitivity. Although it was possible to use the change of inductance to give an increased sensitivity*, it was considered preferable to adopt in the final design a modification which would enable the primary current to be maintained more nearly constant.
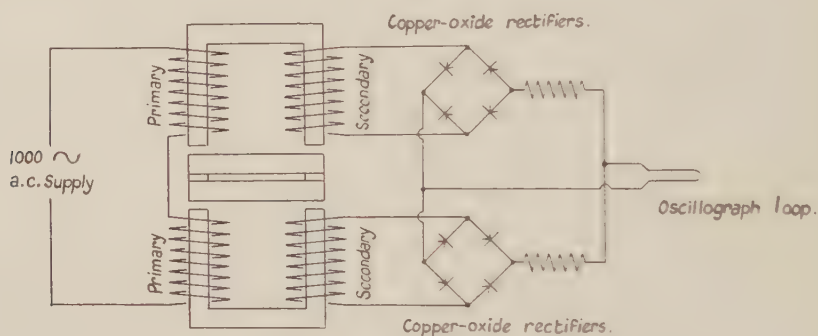
Fig. 2. Electrical connexions of remotely-recording accelerometer†.

This was achieved by a symmetrical disposition of a (double) armature between two similar cores and windings. With this arrangement, for displacements of the armature, which are small compared with the equivalent length of the magnetic path, the increase in the inductance of one winding is balanced by the decrease in that of the other. The use of two magnetic systems combined, as illustrated in figure 2, offers also several additional advantages. The output from the second rectifier can be used to set back electrically the zero of the oscillograph, thus obviating the need for a separate battery and resistance for that purpose; the sensitivity is doubled, since as the output current from one rectifier increases that from the other decreases; and when the two systems are so combined the curvatures in the individual calibration curves tend to neutralize each other, and a curve more nearly linear than either is obtained, thus facilitating the interpretation of the records.

* By connecting in series with the supply a condenser whose reactance was numerically greater than that of the winding, thus giving a decrease of the total primary circuit impedance as the inductance increased.

† Primary and secondary windings are here shown on one limb of each core; in the instrument they are distributed on both limbs.

§ 4. MECHANICAL ASPECTS OF THE PROBLEM

The fundamental form of an accelerometer comprises a mass mounted on a spring. The force necessary to accelerate the mass is transmitted through the spring and the ensuing deflection gives a measure of the force and hence of the acceleration. Since a relative movement of the mass and the system whose acceleration is to be measured is essential for the operation of the recording device, and the deflection of the spring is proportional to the acceleration of the mass itself, it follows that any accelerometer must always be inaccurate to a greater or less degree.

The extent of the error is seen from the following calculation:

Let M be the mass, and k the stiffness of the spring.

Apply an acceleration $A \sin \omega t$ to the spring anchorage, then the displacement of the anchorage is $- (A/\omega^2) \sin \omega t$ and that of the mass itself is $- (A/\omega^2) \sin \omega t - x$, where x is the extension of the spring.

Hence the acceleration of the mass is $A \sin \omega t - d^2x/dt^2$.

The force on the mass is kx.

Hence
$$kx = M (A \sin \omega t - d^2x/dt^2),$$

or
$$(k/M)\, x + d^2x/dt^2 = A \sin \omega t,$$

the solution of which is

$$x = P \sin (\omega_0 t + \phi) + A \sin \omega t/(\omega_0{}^2 - \omega^2),$$

P and ϕ being arbitrary constants while $\omega_0 = \sqrt{(k/M)}$.

The first term corresponds to the free oscillation of the mass on the spring, the second to the extension of the spring under the applied acceleration. Hence the observed deflection $x = (\omega_0{}^2 - \omega^2)^{-1} A \sin \omega t$. Only when ω^2 is small compared with $\omega_0{}^2$ is this closely proportional to the applied acceleration.

The formula shows also that for unit acceleration the extension of the spring is $\omega_0{}^{-2}$; hence as the natural frequency of the system is raised to permit of accurate response to acceleration of higher frequency, the displacement of the mass becomes smaller. For the case when $\omega_0 = 2000$ (corresponding to a natural frequency of just over 300 ∿) the displacement in inches for an acceleration 1 in./sec.^{2} is $2 \cdot 5 \times 10^{-7}$ and for an acceleration equal to g (384 in./sec.^{2}) the displacement is approximately 0·0001 in. The magnitude of this displacement shows how futile any mechanical remote-recording device would be.

In practice it is necessary to provide damping to prevent excessive oscillation at the frequency of the system. This damping modifies the response at lower frequencies, and a treatment of the damped accelerometer is given in the appendix.

§ 5. MECHANICAL DESIGN

The design of the actual instrument presented some difficulties owing to the lack of information relating to the magnitudes and frequencies which had to be measured. American publications suggested that accelerations up to ten or twelve times gravity might be encountered, but no data were available as to the frequency

M, k

A, ω, t

x

P, φ

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of the important components of the acceleration time curve. The only practicable course was to design the instrument to have as high a natural frequency as possible consistent with the desired sensitivity, and to provide in the design for easy variation of this frequency by alteration of the stiffness of the springs.

The following requirements were therefore to be fulfilled in the final design: (1) A range up to 10 g or 15 g. (2) As high a natural frequency as possible. (The experimental apparatus required a displacement of 0·005 in. for full-scale deflection, but it was hoped to improve on this in the final design.) (3) The mass to be mounted so as to have a single degree of freedom. (In practice this means that the natural frequency of vibration along one axis must be much lower than that along either of the others and also than those of oscillation about any axis.) (4) The form of
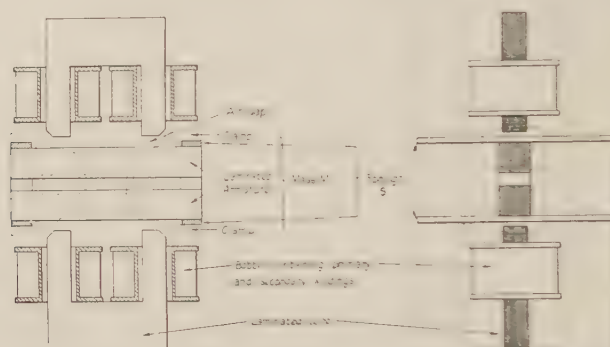
Fig. 3. General arrangement of accelerometer.

the spring to be simple and cheap so as to permit easy substitution of one spring for another of different stiffness. (5) Provision for damping the instrument, and for readily varying the degree of damping since this could not be predetermined. (6) If possible a shape which would move freely inside a cylinder 3 in. in diameter since this would enable the finished instrument to be calibrated by being mounted upon the top of a piston available in the laboratory. (7) A mechanical construction capable of withstanding the vibration to which the instrument would be subjected when mounted upon the rear axle of a vehicle. (8) The weight not to exceed about 10 to 15 lb.

The fundamental form of the design is illustrated in figure 3, the outstanding feature being the use of springs having the form of flat steel rings. Two such springs, their planes separated by a distance roughly equal to ¼ of their diameter, each clamped to the mass at the ends of a diameter, and to the frame of the instrument at the ends of a diameter at right angles, enable the armatures to be mounted with the required single degree of freedom. Their construction is simple and inexpensive, and the symmetrical shape minimizes the likelihood of their moving in the clamp and further results in any such movement having little effect upon the total stiffness.

The complete design is shown in figure 4. The body is divided into two parts 1, 2 spigotted and held together by four bolts 3. Each half is hollowed out to accommodate the laminated cores 4 and windings 5 and turned down to clear the springs 6 which are clamped to the lower half of the body by bolts 7 and fittings 8, 9, 10, 11. Clamps 12 hold the laminated armatures 13 to the springs, a loose packing piece 14 being inserted for constructional purposes. The laminations of the cores are clamped between two plates 15, these in turn being held down by cleats 16 which are made to fit both in the slot in the body and in the gap between the plates 15, thus preventing any side-movement of the cores.
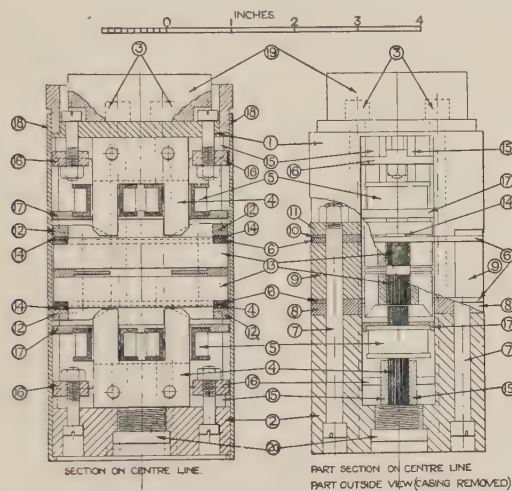
Fig. 4. Mechanical design of accelerometer.

The bobbins containing the windings are clamped with brass plates 17 suitably recessed to fit the bobbin flanges, and having a single slot for the accommodation of both limbs of the cores. Cut in this way, the plate does not link with the main flux, and any eddy currents induced in it are small and due to leakage flux only. The plate fits closely over the limb of the core and into the slot in the body, preventing side-movement of the top of the core. In addition to an oil-tight casing 18 and a terminal block 19, a plug 20 is provided in the base to permit filling the complete instrument with oil to obtain the required damping. Four tapped holes (not shown) are provided in the base for attachment to the axle.

Examination of the drawings shows how the various factors enumerated as affecting the design have been met in the finished instrument.

The choice of the dimensions of the original springs of the instrument presented some difficulty since not only was it difficult to calculate to any useful degree of accuracy the stiffness of the form of spring employed, but in addition the sensitivity of the final electrical arrangement was unknown. Rough calculations of the stiffness of the spring were made, the ring being treated as composed of eight cantilevers; the extra stiffness due to torsion was neglected. Such calculations were

of use in indicating a thickness of 0·1 in. as the maximum thickness likely to be required even for the higher ranges. The thickness specified was 0·07 in. for trial, this value being somewhat thicker than was expected to be necessary since it is clearly a simpler matter to reduce the thickness than make new springs of greater thickness. Fortunately the increased sensitivity of the final apparatus as compared with the experimental apparatus resulted in a suitable range and sensitivity of the instrument with the springs 0·07 in. thick, and these springs have therefore been retained.

The natural frequency of the suspended mass was determined by taking a record while the base was repeatedly struck with a block of rubber. The record so obtained is reproduced in figure 5, from which the natural frequency is seen to be 300 ~.

Fig. 5. Natural frequency of accelerometer (undamped) excited by striking base with block of rubber.
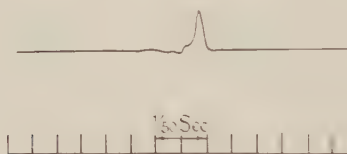
Fig. 6. As figure 5 but casing filled with oil (instrument damped).

Further experiments were made to determine a suitable damping oil; oscillations of the system were excited as before, and it was found that a light mineral oil gave a satisfactory damping-coefficient. The record obtained with the casing filled with this oil is reproduced in figure 6. The ratio of successive swings has fallen to 0·4 as compared with 0·985, when the casing was empty. At the same time the natural frequency has fallen to about 200 or 220 ~, but this figure is difficult to determine with any accuracy owing to the small number of oscillations appearing in the record.

§ 6. ELECTRICAL EQUIPMENT

On the electrical side, the auxiliaries may conveniently be described under three heads: (i) The power supply; (ii) the oscillograph; (iii) the fork and amplifier unit.

(i) The body of the experimental trailer which houses the equipment is divided into two parts, the smaller of which contains a 500-watt 12-volt petrol-electric generating set which supplies the whole of the electrical energy required, and a 12-volt battery which is floated across the lines to steady the supply in the event of slightly irregular running of the engine. (ii) The rear and larger compartment of

the trailer contains the remainder of the apparatus, and is constructed as a dark room so that records can be developed on the road if necessary. The oscillograph used for taking the records is a standard Cambridge three-element instrument, slightly modified to suit the particular conditions, and is mounted upon a sprung table to insulate it from the road shocks. The camera is driven by a $\frac{1}{8}$-h.p. 12-volt motor operated from the power supply. (iii) The appartus for converting the 12-volt supply into the 1000-cycle a.c. supply required by the accelerometer is contained, together with the rectifiers and other details, in an upright cabinet. The three upper shelves are practically identical except for slight changes in the arrangement of the components made to reduce the coupling between the circuits, and each contains a power valve, of the L.S.5a type, which supplies the current to
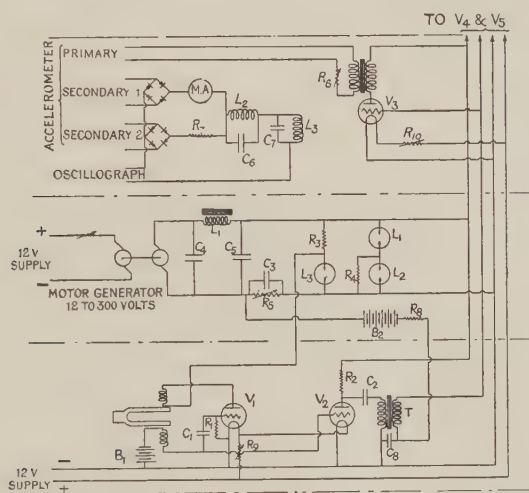
Fig. 7. Wiring diagram of amplifier for 1000 ∼ supply.

the primary winding of the accelerometer through a suitable output transformer. The grid of the valve is excited through a single-valve amplifying stage from the grid coil of a valve-maintained tuning fork, a separate valve being used to sustain the oscillations. These valves together with the fork and coupling units are mounted on a fixed shelf in a cupboard at the bottom of the cabinet. The filaments of the valves are heated from the 12-volt supply through suitable resistances, and h.t. voltage is obtained from a small (permanent-magnet-field) motor-generator with an output of 120 mA. at 300 V.

The complete wiring diagram, excluding two power shelves which are similar to the power shelf shown, is given in figure 7. The grid of the fork-maintaining valve is self-biased with the resistance and condenser, while bias for the amplifying valve is obtained from a small battery B_1. The amplifying valve feeds the grids of the power valves through the resistance-condenser-transformer arrangement R_2, C_2, T.

The output from the motor-generator is smoothed with condensers and a

choke in the conventional manner. To minimize voltage fluctuations due to irregular running of the prime mover two neon lamps L_1 and L_2 connected in series are placed across the h.t. mains and a resistance R_5 is inserted in the negative lead. The voltage drop across this resistance, supplemented by a battery B_2, is used as grid bias for the power stages. To reduce the striking voltage of the two lamps in series, a high resistance R_4 is connected across one of them. The anode supply for the fork-maintaining valve is taken from the h.t. positive main through a resistance R_3, a third neon lamp L_3 being connected in parallel to stabilize further the voltage of the supply to this valve. A resistance R_6 is included in the output circuit to permit the adjustment of the accelerometer current to any desired value.

The rectifiers and other parts in the secondary circuits of the accelerometer are located on the same shelf as the output transformer and power valve. The essentials of the circuit have been shown in figure 2 and the full diagram of connexions is included in figure 7.

As compared with figure 2 the complete circuit exhibits two important modifications. (i) The replacement of one of the resistances of the bridge circuit by a milliammeter. By altering the primary circuit resistance until this meter shows a definite deflection, the conditions existing at the time of calibration can be repeated without reference to the primary circuit conditions. (ii) The addition of two tuned filter circuits L_2, C_6 and L_3, C_7. The output from the rectifiers is not pure d.c. but has a strong superimposed ripple of 2000 $\sim$ together with higher harmonics. It was originally the intention to discriminate between this ripple and the lower frequencies proper to the accelerometer record, by tuning the oscillograph to a frequency of the order of 500–600 $\sim$, under which condition it was hoped that the response to 2000 $\sim$ would be so small as to give no visible ripple in the record. Experiments showed however that there was perceptible ripple of this frequency, and also that owing to lack of perfect symmetry in the rectifiers there was a 1000-cycle component present. To eliminate these two frequencies from the oscillograph record, two tuned filter circuits were placed in series with the instrument as shown in the diagram. Air core inductances of low d.c. resistance were used, tuned with paper condensers of capacities 2 and 4 μF respectively for the 2000 and 1000-cycle circuits. No attempt was made to construct coils of the lowest possible a.c. resistance, because a sharply tuned circuit is undesirable since the ripple is modulated by the acceleration/time wave-form. For the same reason paper condensers were used instead of those having a mica dielectric. With these filter circuits in use there is no perceptible ripple in the record of either 1000 or 2000 $\sim$.

There are three power shelves with rectifiers, filter circuits, etc. Two of these are used for the two accelerometers which have been shown to be necessary for the complete determination of the impact-force on the axle; the third is provided for the load-gauge which is used for determining the load due to the weight of the vehicle and load. For this instrument the same method of remote recording is used as for the accelerometers.

§7. CALIBRATION

Two methods of calibration have been employed, the static and the dynamic.

(i) *Static.* Known weights, increasing in units equal to that of the moving system, were suspended from the moving system of the accelerometer, and a photographic record was exposed for each increment of load. A series of lines giving a scale of acceleration in units of gravity was thus obtained. For convenience in calibration and interpretation this unit of acceleration has been retained throughout. The calibration was repeated for different values of circulation current in the rectifier bridge circuit, and thus a series of curves was obtained permitting the variation of sensitivity over a small range.

(ii) *Dynamic.* The instrument was mounted on the top of a piston actuated by a crank and connecting rod of known dimensions. A photographic record was taken when the crank was rotated at a speed of approximately 350 r.p.m., the actual speed being determined from the time required for one revolution of the crank as deduced

Angular displacement of crank
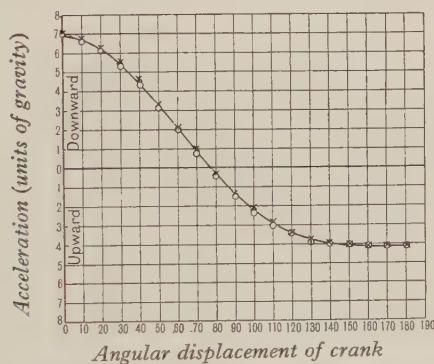
Fig. 8. Calibration test. Curve is plotted from calculated values (not shown). Points are plotted from record obtained from accelerometer and converted into acceleration by use of calibration curve.

from the accelerometer record. Values of acceleration determined from the record so obtained by the use of the calibration curve obtained from static loading were then compared with the calculated values of acceleration of the piston. The agreement is shown in figure 8, in which the curve is plotted from calculated values of acceleration and the isolated points are plotted from the curve of observations. It will be seen that the agreement is very good, the slight difference being due to a small error in locating the top dead-centre position of the crank in the accelerometer record. If one set of points were moved slightly to the right and the other a corresponding distance to the left they would both lie almost exactly on the calculated curve. This readjustment has not been made because the comparison can be made more easily in the figure as drawn than if the points were plotted exactly on the theoretical curve. As the agreement between the two calibrations was so satisfactory, subsequent calibrations rendered necessary by alteration of circuit conditions, etc. were made by the static method only, since this was vastly more convenient in practice.

§8. RESULTS OBTAINED WITH THE INSTRUMENT

As yet the instrument has not been used upon the road but it has been used for some preliminary experiments in the laboratory, with very satisfactory results*. In these experiments a long arm was hinged at one end and carried a wheel fixed at the centre of percussion of the system: the accelerometer was attached to the beam at a position as close to the wheel as possible. The wheel and beam were then raised about the hinge and allowed to fall through known heights, the tyre of the wheel striking a concrete floor on impact. Records of the acceleration of the system were taken during the fall and impact and permitted a comparison to be made of the properties of various types of tyre, etc. It will be seen that the conditions of operation of the instrument were very similar to those under which it will have to operate when upon the road.
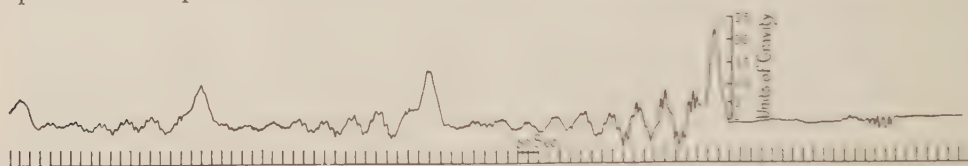
Fig. 9. Successive impacts of a solid-tyred wheel. (Height of fall 4·97 in.)
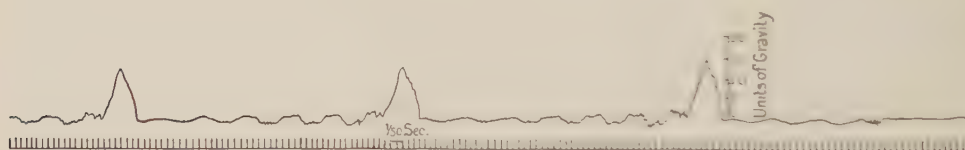
Fig. 10. Successive impacts of a pneumatic tyred wheel. (Height of fall 13·27 in.)

Typical records are reproduced in figures 9 and 10. Referring to figure 9, which is the record obtained from the impact of a solid-tyred wheel falling through 4·97 in., the first break in the curve is due to the release of the beam, and this sets up vibrations of the beam itself which die out after a few hundredths of a second. The displacement of the mean line shows that the accelerometer is falling with an acceleration equal to gravity. The impact of the tyre and ground is easily recognizable, and the shock sets up violent oscillations of the beam which persist during rebound and second fall up to the time of the second impact. This is easily recognizable by the increased amplitude of the oscillations following it.

Figure 10 is a record from a pneumatic tyre falling through 13·27 in. It will be noted that the rate of rise of acceleration and the maximum value attained are both less than the values for the solid-tyred wheel and that the time of contact of the tyre and ground is correspondingly longer. The oscillations of the beam initiated by the impact are less violent also.

* The instrument has subsequently been used upon the road with equally satisfactory results.

§ 9. CONCLUSION

The instrument and equipment described embody a new method of remotely recording small movements which is capable of rapidly and accurately following changes within the limits of the apparatus. In the particular form employed it gives a magnification of the order of 1000 with a range of movement of 0·002 to 0·003 in. at frequencies up to 300 ∼. This latter figure could be increased, with a loss of sensitivity due to an increased supply-frequency and reduced oscillograph-sensitivity.

The object for which this method was devised and to which it has so far been applied is the production of an accelerometer with a natural frequency of 300 ∼ and a range up to 25 *g*. The design of this instrument has been fully described in the present paper, but it is obvious that other applications are possible. In particular the load-gauge required for the completion of the experimental equipment uses the same method; it is described in the subsequent paper*.

§ 10. ACKNOWLEDGMENTS

The author desires to acknowledge the interest taken in the work by the Ministry of Transport, for whose programme of research the instrument was developed, and to tender thanks to them, and to the Director of the National Physical Laboratory, for permission to publish this paper. He wishes also to thank Mr J. H. Hyde, of the National Physical Laboratory, under whose supervision the work was performed, for much helpful advice and criticism, and Mr A. F. C. Brown for considerable assistance in the experimental work with the complete accelerometer.

APPENDIX

FREQUENCY RESPONSE OF AN ACCELEROMETER WITH DAMPING

Suppose the damping to be provided by a dash-pot.

Let y be the displacement of the frame;

  z the displacement of the mass;

  k the stiffness of the spring;

  μ the resistance of dash-pot in units of force per unit velocity; and

  M the mass.

Then the compression of the spring is $y - z$ and the velocity of the dash-pot is $\dot{y} - \dot{z}$.

The force on the mass is

$$(y - z)\,k + (\dot{y} - \dot{z})\,\mu.$$

Equating to $M\ddot{z}$ and reducing, we have

$$\ddot{z} + \frac{\mu}{M}\dot{z} + \frac{k}{M}z = \frac{\mu}{M}\dot{y} + \frac{k}{M}y.$$

* See page 45.

F. Aughtie

To determine the variation of response with frequency put

$$y = A \sin \omega t, \qquad z = C \sin \omega t + D \cos \omega t,$$

so that

$$- C\omega^2 \sin \omega t - D\omega^2 \cos \omega t + \frac{\mu\omega}{M}(C \cos \omega t - D \sin \omega t)$$

$$+ \frac{k}{M}(C \sin \omega t + D \cos \omega t) = \frac{\mu\omega}{M} A \cos \omega t + \frac{k}{M} A \sin \omega t.$$

Equating coefficients of sine and cosine terms and solving for C and D we have

$$C = A \left\{ \frac{k}{M}\left(\frac{k}{M} - \omega^2\right) + \left(\frac{\mu\omega}{M}\right)^2 \right\} \Big/ \left\{ \left(\frac{k}{M} - \omega^2\right)^2 + \left(\frac{\mu\omega}{M}\right)^2 \right\},$$

$$D = A - \frac{\mu\omega}{M}\omega^2 \Big/ \left\{ \left(\frac{k}{M} - \omega^2\right)^2 + \left(\frac{\mu\omega}{M}\right)^2 \right\}.$$

The displacement of the spring, which is the observed quantity,

$$= y - z = (A - C) \sin \omega t - D \cos \omega t = \{(A - C)^2 + D^2\}^{\frac{1}{2}} \sin(\omega t + \phi).$$

On substitution for C and D and evaluation of the coefficient,

$$(\text{Coefficient})^2 = A^2 \frac{\left[\left(\frac{k}{M} - \omega^2\right)^2 + \left(\frac{\mu\omega}{M}\right)^2 - \frac{k}{M}\left(\frac{k}{M} - \omega^2\right) - \left(\frac{\mu\omega}{M}\right)^2\right]^2 + \left[\frac{\mu\omega}{M}\omega^2\right]^2}{\left[\left(\frac{k}{M} - \omega^2\right)^2 + \left(\frac{\mu\omega}{M}\right)^2\right]^2}$$

$$= A^2 \frac{\omega^4}{\left\{\left(\frac{k}{M} - \omega^2\right)^2 + \left(\frac{\mu\omega}{M}\right)^2\right\}}.$$

The applied acceleration $= - A\omega^2 \sin \omega t$. Hence

$$\frac{\text{applied acceleration}}{\text{observed deflection}} = \sqrt{\left\{\left(\frac{k}{M} - \omega^2\right)^2 - \left(\frac{\mu\omega}{M}\right)^2\right\}} \frac{\sin \omega t}{\sin(\omega t + \phi)}.$$

Hence on comparison of amplitudes only,

$$\text{Applied acceleration} = \sqrt{\{(\omega_0^2 - \omega^2)^2 + \alpha^2 \omega^2\}} \times (\text{observed deflection})$$

$$= \sqrt{\left[\left\{1 - \left(\frac{\omega}{\omega_0}\right)^2\right\}^2 + \left\{\frac{\alpha\omega}{\omega_0^2}\right\}^2\right]} \times \omega_0^2 \times (\text{observed deflection}),$$

where $\omega_0^2 = k/M$ and $\alpha = \mu/M$.

But $\omega_0^2 \times$ (observed deflection) is the value of acceleration obtained from static loading, i.e. is the observed acceleration.

Hence the true acceleration is obtained by multiplying the observed values by the factor

$$\sqrt{\left[\left\{1 - \left(\frac{\omega}{\omega_0}\right)^2\right\}^2 + \left\{\frac{\alpha\omega}{\omega_0^2}\right\}^2\right]}.$$

DISCUSSION

For discussion see page 49.

A REMOTE ELECTRICALLY-RECORDING LOAD-GAUGE FOR WHEEL-IMPACT MEASUREMENTS

By F. AUGHTIE, Ph.D., M.Sc.,
of the Engineering Department, National Physical Laboratory

ABSTRACT. The paper is supplementary to the previous paper and describes the load-gauge designed for measuring that component of the wheel-load which arises from the sprung-weight.

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§ 1. INTRODUCTION

IN the preceding paper* it was shown that the impact-force between the wheel of a vehicle and the road can be determined from a knowledge of the linear and angular acceleration of the axle, and the load imposed upon it by the vehicle body. The present paper supplements the one cited and describes the load-gauge designed for measuring that component of wheel-load which arises from the sprung-weight.

There were two alternative positions for the measuring spring: (i) between the chassis and the suspension spring, and (ii) between the suspension spring and the axle. The former position gave greater space for the instrument and considerably simplified the conditions to be met, in that, by being located in the spring shackle, the instrument would be called upon to withstand pure compression loads only. This location was however inferior to (ii) for theoretical reasons. The suspension spring is attached at its centre to the axle and at its ends to the chassis; consequently a portion of the spring moves with the axle, thereby increasing its moment of inertia. The equivalent mass of the spring, i.e. that proportion of the mass which effectively moves with the axle, depends upon the deflection curve of the spring, which in turn is affected by friction between the leaves, etc., and hence can neither be measured nor calculated with any high degree of accuracy. Thus, at the expense of increased complexity and cost, it was considered desirable to insert the measuring unit between the axle and suspension spring if possible.

§ 2. DESIGN OF APPARATUS

This location was achieved by the use of a measuring spring of the form shown in figure 1. The top centre face carries the suspension spring, while the ends are carried from the axle by two split brackets, one on either side. To permit free deflection of the spring the hole in its centre is made of larger diameter than the axle.

* See page 31.

The deflection is observed and recorded by the use of an electrical equipment operating upon the same principle as the accelerometer and attached to the three lower faces of the spring. In this way errors due to slipping or distortion of the loading clamps are entirely eliminated.

The spring was machined from a solid forging of nickel-chrome steel, as it was not considered possible to build up the form required without grave risk of error due to relative movement of the parts. After hardening the spring was ground all over to a depth of approximately 0·005 in. to remove the surface metal and scale.

Under normal load the deflection is approximately 0·005 in. and hence the suspension system of the vehicle is inappreciably affected by the insertion of the instrument. The mass of the axle is increased somewhat, but not to any serious extent. Further, measurements will during the early work be confined to a trailer, the rear axle of which is considerably lighter than that of a self-propelled vehicle.
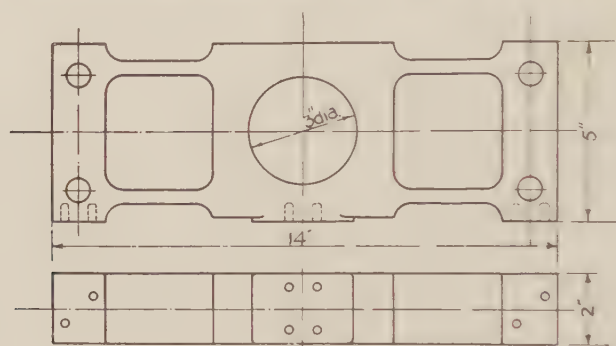
Fig. 1. Measuring spring.

With reference to figure 2, which shows a part section and inverted plan of the complete arrangement, the spring 1 carries an aluminium bridge casting 2 attached by studs 14; this in turn carries the armature 3 mounted on a steel facing 4 and clamped with studs 15 and a pad 5. The cores 6, fitted with wound bobbins, are mounted in the two halves 7, 8 of the body, which are held on to the centre face of the spring by four nickel steel studs 9. The mode of fixing the cores and bobbins is exactly the same as that employed in the accelerometer. Entry of rain and dirt is prevented by a rubber diaphragm 10 which is clamped between the body and a distance piece 11 and permits relative movement of the body and aluminium bridge. Weather-proofing is completed by a sheet-brass cover (not shown) which is fitted to the bottom of the bridge.

The spring itself is carried from the axle 12 by split brackets 13 to which it is held by four bolts 18. The brackets themselves are clamped on the axle by bolts 20.

The suspension spring of the vehicle (not shown) is held down to the top face of the measuring spring 1 by four bolts 16 anchored in a plate 17. Connexion to the windings is made through a terminal block 19; and to permit easy separation of the two halves of the body 7 and 8, plug and socket connectors are interposed in the leads to the winding on the upper half 7.

One feature of the design, of value in view of the fact that the maximum load is not known to any high degree of accuracy, is that in the event of failure of the measuring spring the vehicle will fall on one side by an amount equal to the clearance between the axle and the centre-hole of the spring. Thus although such a mishap would wreck the load-gauge it would not cause disaster to the whole vehicle.

§ 3. CALIBRATION

The stiffness of the spring unmounted was determined from mechanical measurements before the remainder of the instrument was assembled and the nominal value was approximately 550,000 lb./in. On assembly some discrepancy was apparent between the mechanical and electrical measurements; it found that the
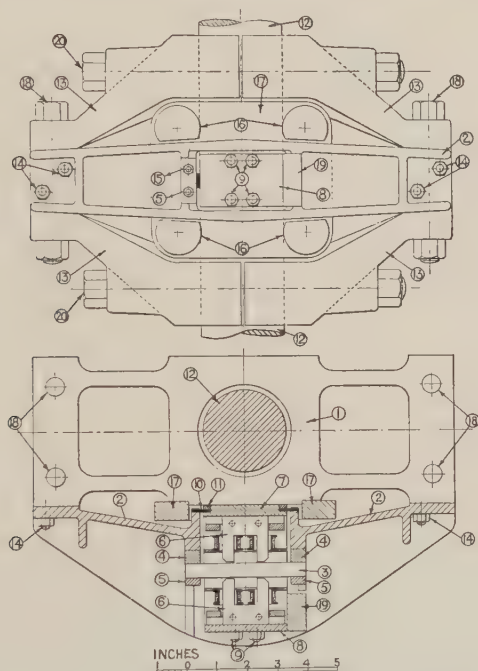
Fig. 2. Complete load-gauge. Above: inverted plan. Below: part section with supporting brackets 13 removed.

load necessary to bring the armature to the central position, as deduced from electrical measurements, was different from that predicted from the measured stiffness of the spring and length of the air-gaps. This disagreement was ultimately found to have three contributory causes: (i) slight differences between the areas of the gaps, (ii) a slight error in the length of the gaps and (iii) a change in the stiffness of the spring when clamped. It so happened that all these causes had cumulative effects, and the discrepancy was therefore more difficult to trace than it would have been if due to a single cause.

A feature of more serious importance was that the calibration curve showed a

hysteresis loop, the width of which amounted to 5 or 6 per cent. of the total load-variation. While this was not so large as to prohibit the use of the instrument—particularly as it would represent a much smaller proportionate error in the wheel load—it was considered too great to be accepted without some attempt at reduction of its magnitude. After several experimental calibrations the hysteresis was found to be purely mechanical and was traced to the influence of the clamping brackets 13, figure 2, and figure 3 shows the calibration curves obtained with the bolts 18 tight and slack respectively.
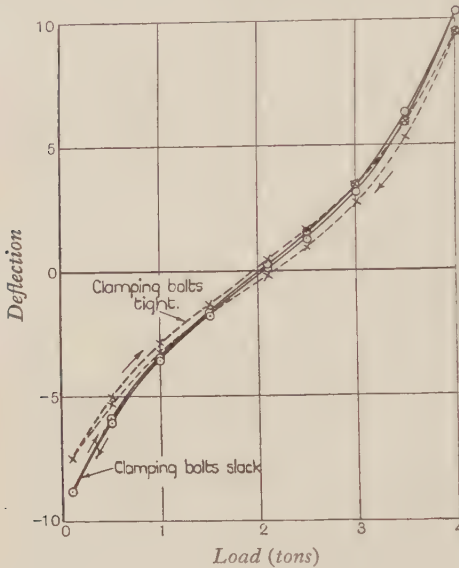
Fig. 3. Preliminary calibration.
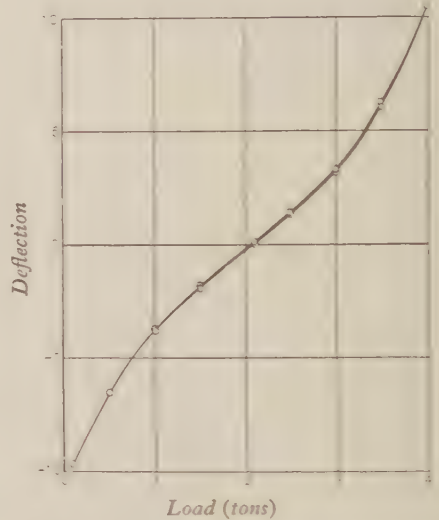
Fig. 4. Calibration with rubber sheet between spring and clamps.

It is clear that when the centre of the spring is depressed under load the ends must approach each other slightly, and while this effect was not overlooked in the design, it was hoped that the magnitude would be so small as not to demand special provision for this relative movement. The calibration showed that this hope was not realized, and while it would have been possible to fit an elastic mounting at one end, a solution, at once much cheaper, simpler, and quite satisfactory, was obtained by interposing between the spring and the faces of the clamps 13 a piece of thin sheet rubber about $\frac{1}{16}$ in. thick. This permitted the slight breathing movement required, and reduced the hysteresis to less than 1 per cent. A layer of tinfoil was inserted between the spring and the rubber to prevent the sulphur from the latter from diffusing into the steel. The final calibration curve is reproduced in figure 4. It may be added that the shape is due to the electrical recording device and that the load-deflection curve of the spring is practically straight over the working range*.

* As in the case of the accelerometer, it proved difficult to predict the stiffness of the spring owing to the large radii left in the corners. The deflection was actually greater than had been anticipated and the air-gaps had therefore to be lengthened. This circumstance is responsible for the increased curvature of the calibration curve as compared with that of the accelerometer.

§4. CONCLUSION

To avoid much repetition in describing the mode of operation, this paper is presented with the previous one instead of being delayed until experimental results have been obtained. It is hoped to publish shortly results obtained with the complete equipment.

§5. ACKNOWLEDGMENTS

DISCUSSION

Mr A. F. DUFTON: The measurement desired is that of the force between one of the back wheels of a vehicle and the road during the impact caused by an irregularity in the road surface. The instruments which the author has described will record only one component of this force. I should like to know whether it is proposed to ignore the other components in the investigation of the destructive action of traffic on a road.
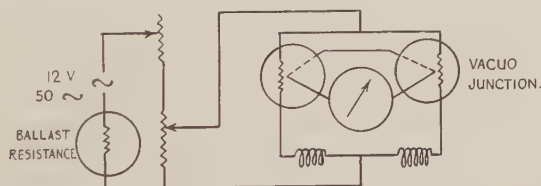
A somewhat analogous method of remotely recording small displacements by electrical means was developed some years ago at the Building Research Station. As in the author's method, displacement of the armature, which in this case is pivoted, increases the inductance of one coil and decreases that of the other. Both coils are supplied with alternating current and in series with each coil is placed the heater of a vacuojunction. The two thermojunctions are opposed and the resultant current is measured on a galvanometer as shown in the accompanying figure. Steadiness in the supply is secured by means of an iron ballast resistance. The calibration of the apparatus showed that the change in galvanometer current was sensibly proportional to the displacement of the armature.

Mr R. S. WHIPPLE: The fact that a small and rapid mechanical movement can be transmitted to an oscillograph opens up a large field of usefulness. Mr E. B. Moullin used a very similar method for recording the variations in the torque of a steamship shaft, the sensitive element of the device being mounted in the tunnel of the ship, and the recording oscillograph in the engine room.

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AUTHOR'S reply: Mr Dufton has drawn attention to a point which was not stressed in the paper, namely, that the instruments record only the force in a vertical direction or, more accurately, in a direction perpendicular to the average contour of the road. It is probably clear that this is all that was attempted. Measurement of forces in a direction parallel to the road (tractive effect and skidding) has been the subject of two other investigations. Since the vertical force is so much larger than the other two it is probable that it plays the principal part in breaking up the road, but now that equipment is available for measuring the various quantities concerned we may hope for definite evidence on this point in the future.

The arrangement of remote recording described by Mr Dufton is of interest and has several points in common with that described in the paper, notably the conversion from a.c. to d.c. and the balanced arrangement of two similar circuits. I imagine, however, that the lag of the thermojunction would render it impossible to follow changes more rapid than, say, $1 \sim$. Also the output power available for operating the indicating instrument is much less than that given by a copper-oxide rectifier.

Mr Whipple refers to the Moullin torsion-meter, the forerunner of many methods of remote recording. Practically all the successful methods of remote recording introduce an a.c. carrier which is modulated by the wave-form to be recorded*.

The differences between the various methods lie in the manner of modulation and demodulation. Apart from the use of the copper-oxide rectifiers, I consider that the most important feature of the method described in the present paper lies in the fact that the equipment was designed as a whole; for instance, the exact form of electrical arrangement employed was determined in part by the nature of the source of a.c. supply, and there are other similar interrelations. I should perhaps mention that a copper-oxide rectifier has also been used in America for remote-indicating purposes†, though only in conjunction with a deflection instrument for visual observation. It is rather curious that, for recording, use was made of an oscillograph with an a.c. field system, since this obviously calls for a very much larger power-supply.

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* Even broadcasting may be included.

† "A magnetic strain gauge," *Proc. Am. Soc. Testing Materials*, **30**, 1041–47. Reference to this paper and the *Annual Report* of the N.P.L. for 1929 and 1930 shows that the ideas were conceived quite independently.

THE BAND SPECTRUM OF ZIRCONIUM OXIDE

By F. LOWATER, Ph.D., F.R.A.S.

Communicated by Prof. A. Fowler, F.R.S., *September* 2, 1931. *Read November* 20, 1931.

ABSTRACT. (i) The spectrum of ZrO has been photographed from λ 2600 to λ 8800, and bands have been found to extend from λ 3200 to λ 7600. (ii) The most prominent bands have been analysed into three systems, all having the same lower electronic state; the blue system α is probably due to a transition $^3\Pi \rightarrow {}^3\Pi$; the yellow system β and the red-infra-red system γ are probably due to transitions $^3\Sigma \rightarrow {}^3\Pi$. (iii) Analysis of the remaining bands is in progress.

§ 1. INTRODUCTION

AFTER publication of the band systems of titanium oxide* it was deemed desirable to analyse the band systems of the analogous molecule, zirconium oxide, the metal of which occurs in the fourth column and fifth row of the periodic table, whereas titanium is in the same column but the fourth row.

Kayser† gives a brief account of the earlier investigations of the band spectrum of compounds of zirconium. Hagenbach and Konen in 1905 observed that the spark spectrum of Zr was full of bands. In 1910 Eder and Valenta published the wave-lengths of thirty-two heads, distributed through the red, orange and yellow regions, λ 6612 to λ 5552, and of one other head at λ 4640, all being degraded toward the red. In the same year Bachem also observed this band spectrum, but published the wave-lengths of only eight heads ranging from λ 6508 to λ 5718 in the visible region; however, he also attributed to the same source five other bands in the ultra-violet, ranging from about λ 3090 to λ 2178, degraded toward the further ultra-violet; he questioned the two of shortest wave-length.

In 1922 Merrill‡ published the results of his investigation of a group of long period variable stars, of which the spectra were in part characterized by strong absorption bands in the blue and red regions; they differed however from stars of class M, since their spectra did not contain the titanium-oxide bands with heads at $\lambda\lambda$ 4761, 4954, 5168 Å. He remarked that the most characteristic feature of their spectra was a complicated structure between λ 4630 and λ 4660, consisting of emission and absorption lines and probably containing one or more band-heads. He identified this part of the spectrum as due to zirconium, by comparing it with that obtained from a zirconium compound in the arc. He concluded that these stars probably formed a third branch of the giant sequence $BAFG$, and were related to stars of class M more nearly than N. The International Astronomical

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* F. Lowater, *Nature*, **123**, 644 (1929); *Proc. Phys. Soc.* **41**, 557 (1929); *Phys. Rev.* **33**, 701 (1929); *Nature*, **123**, 873 (1929); *Astrophys. J.* **70**, 1 (1929).

† *Handbuch d. Spec.* **6**, 864.

‡ *Astrophys. J.* **56**, 457 (1922).

Union (1922) suggested that this class should be designated *S*. Examples of these stars are R Andromedae, T Camelopardalis, R Lyncis, T Sagittarii and R Cygni. The following year Merrill* pointed out an interesting parallel relationship, namely, that while Ti and Zr occupy analogous positions in the periodic table, TiO bands predominate in the *M*-type stars and Zr in a closely related type. He further remarked that stellar observations indicate that the ZrO bands are produced at a higher temperature than the TiO bands.

Merrill's conclusion in regard to temperature was confirmed by Dr A. S. King†, of Mount Wilson Observatory, from his experiments on the bands of Ti and Zr produced in the electric furnace. He found that the bands of TiO appeared at a temperature of about 1900° C.; from a mixture of Ti and Zr in a stream of oxygen TiO bands were very strong at 2200° C., but no ZrO bands appeared; while at 2550° C. the TiO bands were so strong as to make doubtful the existence of the ZrO bands. With Zr alone in the oxygen stream no ZrO bands appeared at a temperature of 2200° C., but at 2550° C. the whole of the ZrO bands appeared, although they were faint compared with those of TiO at that temperature. He concluded that the lower limit of temperature for the appearance of ZrO bands was between 2400° C. and 2500° C. In these experiments King showed that the emitter of the bands was an oxide of Zr.

Not only is the ZrO spectrum characteristic of *S*-type stars, but from his work on "Molecular Spectra in Sunspots," R. S. Richardson‡ obtained evidence that ZrO is included in the sunspot spectrum. He arrived at this conclusion by comparing the sunspot spectrum with the laboratory spectrum of the ZrO band at λ 6474; although he studied other bands, he drew his conclusion from this band because it is very strong and more favourably placed than others for identification, since the sun's spectrum in this region has comparatively few atomic lines and the only other molecular lines belong to a weak TiO band at λ 6478.

Although these ZrO bands are thus well known, no analysis of their structure has yet appeared.

The most prominent bands of the spectrum, and therefore doubtless those most characteristic of the molecule, have been analysed by the writer into three systems of triplets, the systems being designated α, β and γ and the members of the triplets a, b and c. These bands are all degraded toward the infra-red, and therefore the heads must be those of R branches. A second branch is very obvious in systems β and γ. Systems α and γ are analogous to the α and γ systems of TiO, but are displaced toward the shorter wave-lengths, and the intervals between the members of the triplets are greater. System β of ZrO, in the yellow, is a triplet system also and, as will be shown later, has the same lower electronic state as systems α and γ.

Less conspicuous bands, of which about half occur in the regions occupied by systems α, β and γ, have not yet been analysed into a system related to the latter; in the ultra-violet region there is a system differing in general appearance from

* *Publ. Astron. Soc. Pac.* **35**, 218 (1923).
† *Publ. Astron. Soc. Pac.* **36**, 140 (1924).
‡ *Astrophys. J.* **73**, 216 (1931).

α, β and γ, for the bands are much weaker and the triplets not easily recognizable. Comparison of these less conspicuous bands with those of titanium oxide and of hafnium oxide measured by Meggers* and by King† shows that they are not due to these oxides, which might be present as impurities.

Search has been made for vibrational isotope displacements, but with two exceptions none has been found; hence those two are probably accidental coincidences between observed wave-numbers and calculated isotopic displacements.

§ 2. EXPERIMENTAL PROCEDURE

Arcs. The source of the spectrum was chemically pure zirconium oxide on the positive pole of either a Pfund arc, or a similar one with copper poles. A current of about 5 amp. was found most effective in maintaining the bands from the iron arc, but about 9 amp. were required with that of copper. Comparison of this current of 5 amp. with that most effective in maintaining the TiO bands, namely, 2·5 amp., shows that it is in accordance with the results obtained by Dr King in his investigation of these bands by means of the furnace.

Spectrographs. Spectrograms of the red and infra-red regions were obtained by means of a Littrow spectrograph having a dense glass prism of 30°, the dispersion of which ranged from 8·6 Å./mm. at λ 5600 to 35·5 Å./mm. at λ 8700. With this spectrograph panchromatic, kryptocyanin and neocyanin plates were used and the spectrum photographed from λ 4300 to λ 8800. Spectrograms were also taken by means of a 10 ft. Rowland grating in the first order from λ 2600 to λ 6600, in the second order from λ 2600 to λ 3400 and from λ 5030 to λ 6600. With this instrument the photographic plates used were Imperial Ordinary and Ilford panchromatic. Thus with the two instruments the spectrum has been photographed from λ 2600 to λ 8800 and bands have been found to extend from λ 3200 to λ 7600.

Wave-lengths and wave-numbers. The positions of the heads of the bands were measured in the usual manner, and their wave-lengths were determined in international angstroms by reference to those of iron lines, recommended as secondaries by the International Astronomical Union‡ wherever available; and otherwise to wave-lengths published by Meggers and Kiess§. Many iron lines appeared within the band spectrum and were used as standards; they enabled one to avoid errors that might otherwise arise from a slight shift between the band and a comparison spectrum. In cases where these iron lines were not sufficiently strong, the lines of the adjacent comparison spectrum were used, any shift being checked or corrected by means of the iron lines which appeared in both the band and comparison spectra. The wave-lengths of the heads of bands which were hidden by iron lines in the band spectrum were determined from spectrograms taken with the copper arc. The numerous atomic Zr lines included in the spectrum were identified by

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* *Scientific Papers, Bur. Stan.* No. 8,151 (1928).
† *Astrophys. J.* **70**, 113 (1929).
‡ *Trans. Int. Astr. Union*, 3, table 1 (1929).
§ *Scientific Papers, Bur. Stan.* No. 479 (1924).

reference to the tables of the wave-lengths of Zr lines published by King[*] and by Kiess[†]. The wave-numbers of the heads, reduced to vacuum, were obtained by the use of Kayser's[‡] table.

§ 3. STRUCTURE, MULTIPLICITY AND INTENSITY

From the red-infra-red region shown on plate 2 it is evident that the sequences are triple and that system γ is, in the main, formed of three sequences for which $\Delta v = 0$, three for which $\Delta v = +1$ and three for which $\Delta v = -1$. The intensity in the sequences decreases rapidly, but before the sequence ($\Delta v = 0$) of the first member (a) has faded out, that of the second (b) has overlapped it; similarly, the second is overlapped by the third (c). Systems α and β likewise exhibit this triple constitution and overlapping.

This overlapping and complexity of structure make it difficult to follow the sequences to bands of the higher quantum numbers and to form a correct estimate of the relative intensities. It is, however, fairly certain that the intensity of the third member of a triplet is greater than that of the second and that of the second greater than that of the first.

As has been mentioned above, the prominent heads belong to R branches; it will be seen that in the ($v' - v'' = 0$) sequences of the β and γ systems each sub-band has a second branch beginning near the R head, but without rotational analysis it cannot be determined with certainty whether these are Q or P branches. However, close examination of enlarged prints in the second and third orders suggests P branches, for the lines of the R branch, having become comparatively weak, can be easily traced amid the much stronger lines of the second branch; also the lines of the latter are apparently not packed together sufficiently closely to form a Q head. This opinion is formed from the a, b and c members of the 0, 0 band in system γ and the c member of the 0, 0 band in system β. (See plate 2, first, third and fourth strips.) Nevertheless, in system β the b member of the 0, 0 band presents the appearance of a head at the beginning of the second branch. In other cases overlapping of bands renders the character of the second branch obscure, so that its determination must await rotational analysis, which will be a formidable problem even when spectrograms of sufficiently high resolving power have been secured.

§ 4. OBSERVATIONAL DATA

The data of all the true heads are collected in table 1, but held in reserve are some apparent heads which need further investigation to determine their true character; they may be weak heads or mere coincidental grouping of band-lines due to overlapping of several branches. A study of enlarged prints of third-order spectrograms has given much aid in the ascertainment of the constitution of those analysed, and of others included in the table.

[*] *Astrophys. J.* **65**, 86 (1927).
[†] *Scientific Papers, Bur. Stan.* No. 548 (1927).
[‡] *Tabelle d. Schwingungszahlen* (1925).

Column 1 contains the wave-lengths with estimated intensities in parentheses, column 2 the wave-numbers reduced to vacuum. In column 3 are the vibrational quantum numbers, v', v'', with prefixed letters indicating the system, and suffixed letters the branches and members of the triplets. Prefix α indicates the blue system which extends into the violet and yellow, β the yellow and γ the red-infra-red system. R has its usual significance, while X indicates the second branch of undetermined character. The last column, $o-c$, contains the differences between the observed wave-numbers and those calculated from equations (2) for system α, (3) and (4) for system β, and (5) and (6) for system γ.

§ 5. VIBRATIONAL ANALYSIS

The vibrational analysis of the three systems is exhibited in tables 2 to 6, which consist of v' and v'' progressions, arranged in triple form to show the consistency of the triplets.

The wave-numbers of the origins of bands in a system are given by the equation:

$$\nu_0 = \nu_e + \nu_v$$
$$= \nu_e + \{w_e{'} \, (v' + \tfrac{1}{2}) - x_e{'}w_e{'} \, (v' + \tfrac{1}{2})^2 + \ldots\} - \{w_e{''} \, (v'' + \tfrac{1}{2}) - x_e{''}w_e{''} \, (v'' + \tfrac{1}{2})^2$$
$$+ \ldots\} \quad \ldots(1),$$

where v, the vibrational quantum number, takes successive positive integer values,

0, 1, 2, ...;

ν_e is the "system-origin," a wave-number which would arise from an electronic transition alone;

w_e is the frequency (reduced to wave-number units) of vibrations of infinitesimal amplitude about the equilibrium positions of nuclei;

$x_e w_e$ is a coefficient which takes into account the fact that the vibrations are anharmonic, even when $v = 0$;

and the superscripts $'$ and $''$ distinguish quantities pertaining to the upper and lower electronic states respectively.

When band-*head*, instead of band-*origin*, data are used, the equation which applies has the same form but slightly different coefficients, and an additional term in $(v' + \tfrac{1}{2})(v'' + \tfrac{1}{2})$. No reliable determination of the coefficients of this term was possible in the present band systems.

Equations (2) to (6) have been deduced from the data in tables 2 to 6 respectively. As the separation of the triplets changes progressively as v'' increases, the coefficients were calculated separately for each of the three members of the triplets; otherwise unduly large residuals $(o - c)$ would have been obtained. However, for system α and the second branch of system β insufficient data were available to make this course feasible.

Equations for the three systems follow in succession.

For the R branches in system α

$$(a) \quad \nu_\text{head} = \begin{cases} 21698\cdot1 \\ 21614\cdot3 \\ 21601\cdot4 \end{cases} \begin{aligned} &+ \{820\cdot58\,(v' + \tfrac{1}{2}) - 3\cdot306\,(v' + \tfrac{1}{2})^2\} \\ &- \{937\cdot20\,(v'' + \tfrac{1}{2}) - 3\cdot346\,(v'' + \tfrac{1}{2})^2\} \quad \ldots(2). \end{aligned}$$

For the R branches in system β

$$(a) \quad \nu_\text{head} = 18053\cdot0 + \{846\cdot20\,(v' + \tfrac{1}{2}) - 3\cdot675\,(v' + \tfrac{1}{2})^2\} \\ - \{937\cdot47\,(v'' + \tfrac{1}{2}) - 3\cdot633\,(v'' + \tfrac{1}{2})^2\} \quad \ldots(3a),$$

$$(b) \quad \nu_\text{head} = 17806\cdot0 + \{844\cdot26\,(v' + \tfrac{1}{2}) - 3\cdot642\,(v' + \tfrac{1}{2})^2\} \\ - \{936\cdot11\,(v'' + \tfrac{1}{2}) - 3\cdot931\,(v'' + \tfrac{1}{2})^2\} \quad \ldots(3b),$$

$$(c) \quad \nu_\text{head} = 17529\cdot2 + \{845\cdot71\,(v' + \tfrac{1}{2}) - 3\cdot594\,(v' + \tfrac{1}{2})^2\} \\ - \{937\cdot10\,(v'' + \tfrac{1}{2}) - 3\cdot717\,(v'' + \tfrac{1}{2})^2\} \quad \ldots(3c).$$

For the unidentified branches, X, in system β

$$(a) \quad \nu_\text{head} = \begin{cases} 18038\cdot7 \\ 17790\cdot1 \\ 17510\cdot7 \end{cases} \begin{aligned} &+ \{846\cdot28\,(v' + \tfrac{1}{2}) - 3\cdot409\,(v' + \tfrac{1}{2})^2\} \\ &- \{937\cdot17\,(v'' + \tfrac{1}{2}) - 3\cdot417\,(v'' + \tfrac{1}{2})^2\} \ldots(4). \end{aligned}$$

For the R branches in system γ

$$(a) \quad \nu_\text{head} = 16088\cdot9 + \{855\cdot85\,(v' + \tfrac{1}{2}) - 3\cdot183\,(v' + \tfrac{1}{2})^2\} \\ - \{937\cdot00\,(v'' + \tfrac{1}{2}) - 3\cdot383\,(v'' + \tfrac{1}{2})^2\} \quad \ldots(5a),$$

$$(b) \quad \nu_\text{head} = 15791\cdot7 + \{852\cdot50\,(v' + \tfrac{1}{2}) - 3\cdot025\,(v' + \tfrac{1}{2})^2\} \\ - \{935\cdot34\,(v'' + \tfrac{1}{2}) - 3\cdot106\,(v'' + \tfrac{1}{2})^2\} \quad \ldots(5b),$$

$$(c) \quad \nu_\text{head} = 15483\cdot8 + \{853\cdot27\,(v' + \tfrac{1}{2}) - 3\cdot200\,(v' + \tfrac{1}{2})^2\} \\ - \{935\cdot05\,(v'' + \tfrac{1}{2}) - 3\cdot200\,(v'' + \tfrac{1}{2})^2\} \quad \ldots(5c).$$

For the unidentified branches, X, in system γ

$$(a) \quad \nu_\text{head} = 16075\cdot0 + \{856\cdot99\,(v' + \tfrac{1}{2}) - 3\cdot225\,(v' + \tfrac{1}{2})^2\} \\ - \{939\cdot62\,(v'' + \tfrac{1}{2}) - 3\cdot683\,(v'' + \tfrac{1}{2})^2\} \quad \ldots(6a),$$

$$(b) \quad \nu_\text{head} = 15782\cdot5 + \{856\cdot73\,(v' + \tfrac{1}{2}) - 3\cdot562\,(v' + \tfrac{1}{2})^2\} \\ - \{940\cdot55\,(v'' + \tfrac{1}{2}) - 3\cdot867\,(v'' + \tfrac{1}{2})^2\} \quad \ldots(6b),$$

$$(c) \quad \nu_\text{head} = 15467\cdot4 + \{857\cdot18\,(v' + \tfrac{1}{2}) - 3\cdot333\,(v' + \tfrac{1}{2})^2\} \\ - \{939\cdot70\,(v'' + \tfrac{1}{2}) - 3\cdot675\,(v'' + \tfrac{1}{2})^2\} \quad \ldots(6c).$$

The coefficients of $(v'' + \tfrac{1}{2})$ in equations (2) to (6) show that all three systems have the same lower electronic state; as in the analogous case of TiO, this is probably the ground state of the molecule and is a $^3\Pi$ state. In order to arrive at a plausible interpretation of the three upper electronic states, it is necessary to make a few assumptions. Firstly, noting that in ZrO the triplet separations in system γ are greater than in the two other systems, we may assume that in the upper electronic

states of this system the three sub-states are very near together and that this is a $^3\Sigma$ state. This is in analogy with the red system γ of TiO, which is also ascribed to a $^3\Sigma \rightarrow {}^3\Pi$ transition. Assuming that the separations are zero in a $^3\Sigma$ state, we have for those in the ground state of ZrO

$$R_a - R_b = 292 \cdot 1 \quad \text{and} \quad R_b - R_c = 313 \cdot 4,$$

or

$$X_a - X_b = 293 \cdot 1 \quad \text{and} \quad X_b - X_c = 314 \cdot 5.$$

Secondly, it may be assumed that the blue system α of ZrO arises, as that of TiO, from a transition $^3\Pi \rightarrow {}^3\Pi$, and that the relatively small separations observed,
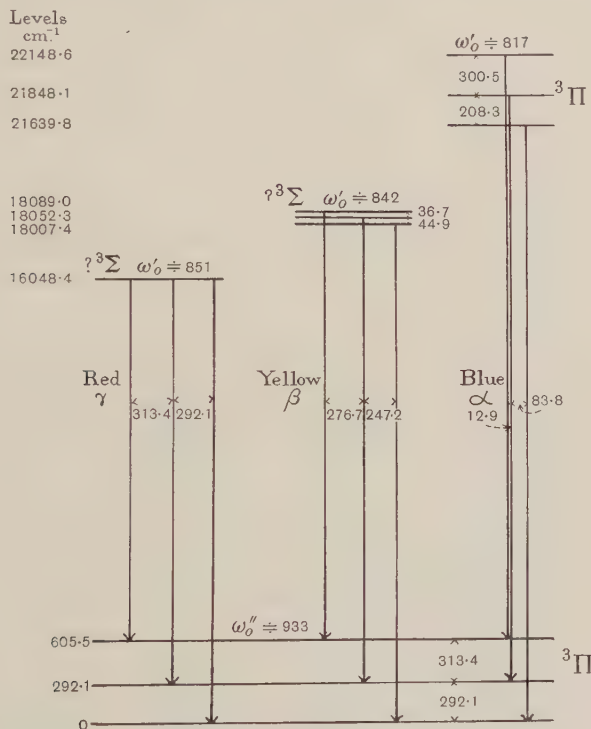
Fig. 1. Provisional energy-level diagram for ZrO.

namely, $R_a - R_b = 83 \cdot 8$ and $R_b - R_c = 12 \cdot 9$, are the differences between the separations in the upper and lower $^3\Pi$ states; in this way we obtain the separations in the upper $^3\Pi$ state as

$$292 \cdot 1 - 83 \cdot 8 = 208 \cdot 3 \quad \text{and} \quad 313 \cdot 4 - 12 \cdot 9 = 300 \cdot 5,$$

less of course than the separations in the lower $^3\Pi$ state. With these assumptions for systems γ and α we obtain, from the observed separations in the yellow system β, its separations in the upper state; thus from $R_a - R_b$ and $R_b - R_c$ we get

$$292 \cdot 1 - 247 \cdot 2 = 44 \cdot 9 \quad \text{and} \quad 313 \cdot 4 - 276 \cdot 7 = 36 \cdot 7,$$

or from $X_a - X_b$ and $X_b - X_c$ we get

$$293 \cdot 1 - 248 \cdot 6 = 44 \cdot 5 \quad \text{and} \quad 314 \cdot 5 - 279 \cdot 4 = 35 \cdot 1, \quad \text{respectively.}$$

Although not negligible, these separations are so much smaller than those in either of the $^3\Pi$ states, that they suggest a $^3\Sigma$ state for system β. The interpretation of systems γ and β as due to transitions $^3\Sigma \to {}^3\Pi$ is in conformity with the fact that other heads are observed in these bands and that these may be heads of Q branches.

These tentative assumptions and assignments are shown in a provisional energy-level diagram in figure 1. It would have been more in conformity with other band spectra had the separations in the upper $^3\Pi$ state been more nearly equal to one another and had those in the upper (? $^3\Sigma$) state of system β been still smaller. However, this may be the case, since the separations are based on the assumption that the upper (? $^3\Sigma$) state of system γ has zero, and not merely small separations*. Repeated attempts on the analysis, based on confidence in the separation of the triplets forming system γ (evident in plate 2), have however yielded no separations which appear to be more satisfactory.

§ 6. ACKNOWLEDGMENTS

§ 7. DESCRIPTION OF PLATES

All strips except the tenth are grating spectrograms of the first order, covering a range of 2280 Å. namely, λ 4300 to λ 6580. The tenth strip is a prism spectrogram taken with a kryptocyanin plate and includes the region from λ 6200 to λ 7700. Unbroken lines ——— show the sequences of system α on plate 1, broken lines — — — those in system β on plates 1 and 2, and unbroken lines ——— those in system γ on plate 2. To avoid overcrowding, the X heads are not marked on the plates, nor are the rather outlying R heads.

Notes to table 2. (i) Data for what appear to be R heads of further bands of system α are:

$$\begin{cases} a \\ b \\ c \end{cases} \quad \begin{cases} 18482 \cdot 4 \\ 18400 \cdot 3 \\ 18387 \cdot 5 \end{cases} \quad \begin{cases} 18387 \cdot 5 \\ 18302 \cdot 5 \\ 18291 \cdot 7 \end{cases} \quad \begin{cases} 18291 \cdot 7 \\ 18208 \cdot 0 \end{cases} \quad \begin{cases} 20709 \cdot 4 \\ 20624 \cdot 8 \\ 20614 \cdot 0 \end{cases}$$

all of which give only small values of $(o-c)$, the greatest being $+ 1 \cdot 6$.

(ii) A satisfactory explanation of the incompleteness of this table is not forthcoming; it may be that the missing band-heads are merely obscured by overlapping branches. Attention may be drawn to the similar character of the progressions of system α of TiO, as analysed by Christy; he found the blue-green system to consist of eleven sequences containing thirty-nine bands, but of these only ten had all three components of the triplets, three had two and twenty-six had only one component. Table 2 might be extended if, as in the case of TiO, confidence could be placed in the assignment of observed heads to positions in the table involving quantum numbers from 5 to 9; then the system would consist of ten sequences containing thirty bands, seven having three components of the triplets, seven having two, and sixteen only one component. Both spectra, ZrO and TiO, possess some bands not yet included in the analyses.

* It may be well to emphasize that these deductions are on band-head data only.

Bands of zirconium oxide from λ 4300 to λ 5600.

Bands of zirconium oxide from $\lambda 5500$ to $\lambda 7700$.
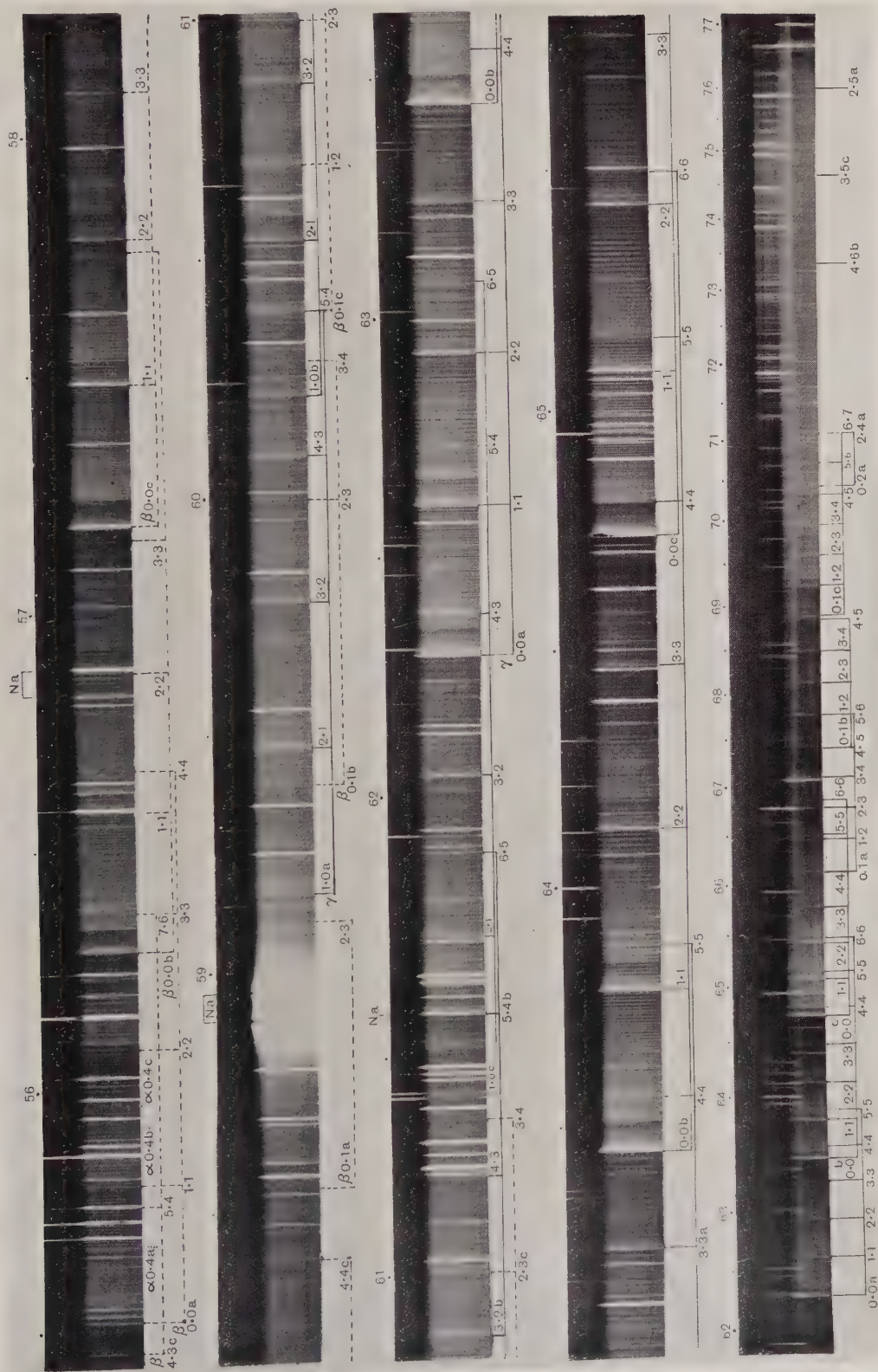
Table 1

λ	ν	v', v''	o − c
3472·43 (5)	28790·1		
3491·84 (5)	28630·0		
3491·97 (6)	28629·0		
3506·22 (5)	28512·6		
3508·24 (5)	28496·2		
3511·93 (2)	28466·3		
3515·79 (2)	28435·0		
3589·81 (3)	27848·7		
3593·07 (2)	27823·7		
3598·92 (2)	27783·6		
3682·43 (6)	27148·3		
3726·31 (3)	26828·6		
4257·11 (2)	23483·0		
4287·81 (2)	23315·1		
4313·32 (3)	23177·5	α 2, 0 R_b	+ 0·3
4340·18 (2)	23034·1	α 3, 1 R_c	− 0·6
4361·95 (3)	22919·1	α 4, 2 R_b	+ 1·2
4363·04 (2)	22913·4		
4364·38 (4)	22906·4	α 4, 2 R_c	− 1·4
4365·14 (4)	22902·4		
4368·16 (3)	22886·5		
4375·08 (3)	22850·3		
4452·21 (1)	22454·5	α 1, 0 R_a	+ 0·7
4460·39 (2)	22413·3		
4469·38 (8)	22368·6	α 1, 0 R_b	− 1·4
4471·53 (10)	22357·4	α 1, 0 R_c	+ 0·3
4474·87 (3)	22340·8		
4493·79 (7)	22246·7	α 2, 1 R_b	− 0·1
4496·24 (8)	22234·6	α 2, 1 R_c	+ 0·7
4519·26 (6)	22121·3	α 3, 2 R_b	− 2·5
4521·26 (6)	22111·5	α 3, 2 R_c	− 0·4
4534·41 (8)	22047·4		
4542·55 (7)	22007·9		
4545·15 (5)	21995·3		
4593·53 (1)	21763·7		
4610·25 (2)	21684·7		
4619·82 (15)	21639·8	α 0, 0 R_a	
4637·79 (18)	21556·0	α 0, 0 R_b	
4640·56 (20)	21543·1	α 0, 0 R_c	
4644·68 (10)	21524·0	α 1, 1 R_a	+ 0·7
4716·86 (2)	21194·6	α 3, 3 R_c	+ 0·8
4736·90 (12)	21105·0		
4740·49 (3)	21089·0		
4769·35 (2)	20961·4		
4771·25 (2)	20953·0		
4797·16 (1)	20839·8		
4798·09 (1)	20835·8		
4827·38 (3)	20709·4	? α 8, 8 R_a	− 0·3
4827·52 (8)	20708·5	α 0, 1 R_a	− 0·8
		{ α 0, 1 R_b	− 0·7
4847·19 (5)	20624·8	{ ? α 8, 8 R_b	− 1·1
4849·72 (3)	20614·0	? α 8, 8 R_c	+ 1·0
4850·15 (6)	20612·2	α 0, 1 R_c	− 0·4
4863·08 (4)	20557·4		
4873·53 (1)	20513·3	α 1, 2 R_b	− 2·4
		{ α 1, 2 R_c	− 0·4
4876·12 (3)	20502·4	{ β 3, 0 R_a	+ 0·5
4899·76 (3)	20403·5	α 2, 3 R_b	− 2·4
4902·47 (3)	20392·2	α 2, 3 R_c	− 0·8
5103·85 (1)	19587·6	α 1, 3 R_c	+ 1·9
5156·34 (2)	19388·2		
5185·00 (7)	19281·1		
5212·19 (4)	19180·5	α 0, 3 R_a	+ 0·5
5298·26 (2)	18868·9	β 1, 0 R_a	0·0
5304·63 (1)	18846·2		
5305·96 (2)	18841·5		
5308·42 (2)	18832·8	β 1, 0 X_a	0·0
5322·03 (2)	18784·6	α 0, 3 R_b	0·0

Table 1 (*continued*)

λ	ν	v′, v″			o − c
5332·50 (2)	18747·1		β 2, 1 R_a		− 0·4
5360·70 (3)	18649·1		β 3, 2 R_a		+ 0·3
5364·34 (2)	18636·5		β 3, 2 X_a		− 0·9
5375·68 (2)	18597·1		β 1, 0 R_b		− 0·1
5377·26 (2)	18591·7				
5379·50 (3)	18583·9		β 1, 0 X_b		− 0·3
5389·25 (2)	18550·3		β 4, 3 R_a		+ 0·4
5390·36 (2)	18546·5				
5392·96 (2)	18537·6		β 4, 3 X_a		− 2·1
5404·35 (4)	18498·5		β 2, 1 R_b		− 0·2
5407·14 (3)	18488·9				
5408·16 (2)	18485·5		β 2, 1 X_b		− 1·0
5409·05 (3)	18482·4	{ ? α 3, 6 R_c			− 0·1
		{ ? α 4, 7 R_a			− 0·6
5422·04 (1)	18438·1		β 5, 4 X_a		− 1·0
5433·18 (3)	18400·3	{ ? α 4, 7 R_b			+ 1·1
			β 3, 2 R_b		− 0·4
5436·96 (8)	18387·5	{ ? α 4, 7 R_c			+ 1·2
		{ ? α 5, 8 R_a			+ 0·7
			β 3, 2 X_b		− 1·3
5439·40 (7)	18379·2			γ 5, 2 R_a	+ 0·7
5456·49 (8)	18321·7		β 1, 0 R_c		− 0·3
5461·61 (5)	18304·3		β 1, 0 X_c		− 0·5
5462·21 (5)	18302·5	{ ? α 5, 8 R_b			− 0·5
			β 4, 3 R_b		+ 0·9
5465·44 (6)	18291·7	{ ? α 5, 8 R_c			+ 1·6
		{ ? α 6, 9 R_a			+ 1·0
			β 4, 3 X_b		+ 0·6
5465·95 (4)	18290·0				
5485·71 (6)	18224·4		β 2, 1 R_c		+ 0·7
5490·28 (3)	18209·0	{ ? α 6, 9 R_b			+ 2·1
			β 2, 1 X_c		+ 1·9
5491·67 (6)	18204·4		β 5, 4 R_b		− 2·0
5502·84 (3)	18167·4				
5515·33 (6)	18126·3		β 3, 2 R_c		+ 0·7
5519·56 (3)	18112·4	{	β 3, 2 X_c		+ 3·0
		{	β 6, 5 R_b		+ 2·2
5538·80 (5)	18049·5				
5539·35 (6)	18047·7				
5545·16 (6)	18028·8		β 4, 3 R_c		+ 1·0
5547·42 (2)	18021·2				
5551·74 (10)	18007·4		β 0, 0 R_a		
5553·10 (10)	18003·0				
5556·09 (3)	17993·3		β 0, 0 X_a		
5562·67 (2)	17972·0				
5566·93 (3)	17958·3	{ α 0, 4 R_a			− 0·4
		{		γ 3, 0 X_c	+ 0·6
5575·74 (5)	17929·9		β 5, 4 R_c		− 0·2
5580·09 (5)	17915·9		β 1, 1 R_a		− 0·2
5581·74 (6)	17910·6				
5584·56 (2)	17901·6		β 1, 1 X_a		− 0·9
5592·53 (2)	17876·0	α 0, 4 R_b			+ 1·9
5596·49 (2)	17863·4	{ α 0, 4 R_c			+ 0·3
		{		γ 4, 1 R_c	+ 0·3
5608·87 (2)	17824·0		β 2, 2 R_a		− 0·6
5610·05 (10)	17820·2				
5612·61 (3)	17812·1		β 2, 2 X_a		+ 0·5
5623·97 (2)	17776·1				
5629·00 (12)	17760·2	{	β 0, 0 R_b		
		{		γ 5, 2 R_c	− 2·0
5629·53 (14)	17758·5				
5633·92 (6)	17744·7		β 0, 0 X_b		
5634·88 (5)	17741·7			γ 2, 0 R_a	+ 0·7
5636·97 (5)	17735·1	{	β 3, 3 R_a		+ 2·0
		{	β 7, 6 R_c		− 0·6
5658·13 (9)	17668·8		β 1, 1 R_b		− 0·2
5663·06 (4)	17653·4		β 1, 1 X_b		− 0·5
5664·19 (3)	17649·9				
5665·52 (3)	17645·7			γ 3, 1 R_a	− 1·9

Table 1 (*continued*)

λ	ν	v', v''	$o-c$
5666·80 (4)	17641·8	β 4, 4 R_a	+ 0·3
5670·67 (2)	17629·7	β 4, 4 X_a	− 0·2
5681·08 (4)	17597·4		
5687·43 (5)	17577·4	β 2, 2 R_b	− 0·8
5692·30 (4)	17562·6	β 2, 2 X_b	− 0·4
5716·61 (1)	17488·0	β 3, 3 R_b	− 0·2
5718·11 (20)	17483·5	β 0, 0 R_c	
5724·05 (11)	17465·3	β 0, 0 X_c	
5748·14 (16)	17392·1	β 1, 1 R_c	− 0·2
5753·80 (6)	17375·0	β 1, 1 X_c	+ 0·5
5776·05 (1)	17308·1	β 5, 5 R_b	− 1·5
5778·46 (10)	17300·8	β 2, 2 R_c	− 0·7
5783·83 (3)	17284·8	β 2, 2 X_c	+ 1·2
5809·18 (7)	17209·4	β 3, 3 R_c	− 1·4
5814·49 (4)	17193·7	β 3, 3 X_c	+ 1·0
5839·79 (2)	17119·2	β 4, 4 R_c	− 1·2
5854·30 (1)	17076·7	β 0, 1 R_a	− 0·5
5860·09 (12)	17059·8		
5867·98 (7)	17036·9	γ 3, 1 R_c	+ 1·4
5908·52 (10)	16920·0		
5911·45 (4)	16911·6	β 2, 3 R_a	+ 2·6
5916·44 (4)	16897·4	γ 1, 0 R_a	− 0·5
5917·67 (4)	16893·9	β 2, 3 X_c	− 1·0
5923·33 (4)	16877·7		
5939·08 (4)	16833·0	β 0, 1 R_b	+ 1·0
5946·99 (6)	16810·6	γ 2, 1 R_a β 3, 4 X_a	− 0·2 − 0·3
5951·85 (3)	16796·8	γ 2, 1 X_a	+ 0·7
5977·68 (6)	16724·3	γ 3, 2 R_a	+ 0·2
5983·29 (4)	16708·6	γ 3, 2 X_a	+ 0·4
5999·44 (3)	16663·6	β 2, 3 R_b	− 2·2
6008·50 (4)	16638·5	γ 4, 3 R_a	+ 0·7
6014·46 (2)	16622·0	γ 4, 3 X_a	− 0·6
6021·29 (6)	16603·1	γ 1, 0 R_b	+ 0·4
6026·04 (2)	16590·1	γ 1, 0 X_b	+ 0·7
6028·37 (1)	16583·6	β 3, 4 R_b	+ 0·1
6039·13 (4)	16554·1	β 0, 1 R_c γ 5, 4 R_a	+ 0·3 + 2·2
6053·82 (5)	16513·9	γ 2, 1 R_b	− 0·1
6059·33 (2)	16498·8	γ 2, 1 X_b	− 1·2
6070·01 (6)	16469·9	β 1, 2 R_c	− 0·2
6086·85 (5)	16424·3	γ 3, 2 R_b	− 1·2
6091·53 (3)	16411·7	γ 3, 2 X_b	+ 1·4
6099·85 (3)	16389·3	β 2, 3 R_c	+ 2·6
6020·10 (3)	16335·1	γ 4, 3 R_b	− 2·0
6125·36 (1)	16321·1	γ 4, 3 X_b	− 0·1
6131·54 (1)	16304·6	β 3, 4 R_c	+ 1·2
6137·22 (4)	16289·5	γ 1, 0 R_c	− 0·3
6153·92 (8)	16245·3		
6170·20 (4)	16202·5	γ 2, 1 R_c	+ 1·0
6175·18 (2)	16189·4	γ 2, 1 X_c	+ 1·2
6188·01 (4)	16155·8	β 0, 2 R_a	− 0·2
6188·67 (1)	16154·1		
6200·88 (1)	16122·8		
6203·94 (4)	16114·3	γ 3, 2 R_c	+ 1·0
6210·20 (4)	16098·1	γ 3, 2 X_c	− 2·3
6222·78 (2)	16065·6		
6229·40 (18)	16048·4	γ 0, 0 R_a	
6235·10 (6)	16033·8	γ 0, 0 X_a	
6238·92 (6)	16024·0	γ 4, 3 R_c	− 1·2
6260·89 (16)	15967·8	γ 1, 1 R_a	+ 0·2
6266·52 (6)	15953·4	γ 1, 1 X_a	+ 1·3
6272·98 (4)	15937·0	γ 5, 4 R_c	− 0·1
6292·79 (14)	15886·6	γ 2, 2 R_a	− 0·7
6298·95 (4)	15871·2	γ 2, 2 X_a	− 0·1
6308·70 (2)	15846·8	γ 6, 5 R_c	− 2·0
6312·46 (1)	15837·3	β 1, 3 R_b	+ 1·2
6317·63 (2)	15824·3		
6324·33 (6)	15807·6	γ 3, 3 R_a	+ 0·2

Table 1 (*continued*)

λ	ν	v', v''	$o-c$
6331·35 (1)	15790·1	$\gamma\,3, 3\ X_a$	− 1·3
6344·91 (18)	15756·3	$\gamma\,0, 0\ R_b$	
6351·23 (6)	15740·7	$\gamma\,0, 0\ X_b$	
6356·26 (6)	15728·2	$\gamma\,4, 4\ R_a$	+ 0·4
6362·51 (5)	15712·7	$\gamma\,4, 4\ X_a$	+ 0·3
6378·32 (16)	15673·8	$\gamma\,1, 1\ R_b$	+ 0·1
6384·57 (4)	15658·4	$\gamma\,1, 1\ X_b$	+ 0·9
6387·84 (4)	15650·4	$\gamma\,5, 5\ R_a$	+ 1·7
6394·07 (5)	15635·2	$\gamma\,5, 5\ X_a$	+ 0·8
6395·99 (4)	15630·4	$\beta\,0, 2\ R_c$	− 1·2
6412·29 (12)	15590·8	$\gamma\,2, 2\ R_b$	− 0·4
6419·28 (2)	15573·8	$\gamma\,2, 2\ X_b$	− 1·1
6446·52 (3)	15508·0	$\gamma\,3, 3\ R_b$	− 0·8
6452·50 (2)	15493·6	$\gamma\,3, 3\ X_b$	+ 0·6
6473·67 (20)	15442·9	$\gamma\,0, 0\ R_c$	
6480·71 (4)	15426·2	{ $\gamma\,0, 0\ X_c$ $\gamma\,4, 4\ R_b$	− 0·4
6508·15 (18)	15361·1	$\gamma\,1, 1\ R_c$	+ 0·1
6515·15 (4)	15344·6	{ $\gamma\,1, 1\ X_c$ $\gamma\,5, 5\ R_b$	+ 0·2 0·0
6542·98 (10)	15279·4	$\gamma\,2, 2\ R_c$	+ 0·1
6549·72 (2)	15263·6	{ $\gamma\,2, 2\ X_c$ $\gamma\,6, 6\ R_b$	+ 0·4 + 1·0
6578·20 (4)	15197·5	$\gamma\,3, 3\ R_c$	0·0
6584·92 (2)	15182·0	$\gamma\,3, 3\ X_c$	− 0·7
6613·07 (3)	15117·4	{ $\gamma\,4, 4\ R_c$ $\gamma\,0, 1\ R_a$	+ 1·8 − 0·8
6619·64 (1)	15102·4	{ $\gamma\,4, 4\ X_c$ $\gamma\,0, 1\ X_a$	− 0·6 + 0·8
6645·26 (2)	15044·2	$\gamma\,1, 2\ R_a$	0·0
6649·52 (2)	15034·5	$\gamma\,5, 5\ R_c$	+ 0·5
6678·00 (5)	14970·4	$\gamma\,2, 3\ R_c$	− 0·2
6686·58 (2)	14951·2	$\gamma\,6, 6\ R_c$	− 1·0
6693·95 (1)	14934·8		
6710·87 (2)	14897·1	$\gamma\,3, 4\ R_a$	− 0·4
6717·99 (2)	14881·3	$\gamma\,3, 4\ X_a$	0·0
6742·49 (4)	14827·2	{ $\gamma\,4, 5\ R_a$ $\gamma\,0, 1\ R_b$	+ 4·5 0·0
6751·52 (1)	14807·4	$\gamma\,0, 1\ X_b$	− 0·5
6777·21 (4)	14751·3	{ $\gamma\,5, 6\ R_a$ $\gamma\,1, 2\ R_b$	− 1·0 + 0·5
6785·86 (1)	14732·5	$\gamma\,1, 2\ X_b$	+ 0·1
6812·53 (3)	14674·8	$\gamma\,2, 3\ R_b$	+ 0·4
6820·93 (1)	14656·7	$\gamma\,2, 3\ X_b$	− 0·9
6847·33 (3)	14600·2		
6848·92 (3)	14596·8	$\gamma\,3, 4\ R_b$	− 1·5
6854·80 (1)	14584·3	$\gamma\,3, 4\ X_b$	+ 1·0
6884·80 (2)	14520·8	$\gamma\,4, 5\ R_b$	− 1·5
6887·79 (4)	14514·5	$\gamma\,0, 1\ R_c$	+ 0·3
6897·81 (1)	14493·4	$\gamma\,0, 1\ X_c$	− 0·4
6923·73 (5)	14439·1	$\gamma\,1, 2\ R_c$	+ 0·3
6931·83 (5)	14422·2	$\gamma\,1, 2\ X_c$	+ 2·8
6959·90 (6)	14364·1	$\gamma\,2, 3\ R_c$	+ 0·7
6968·93 (2)	14345·5	$\gamma\,2, 3\ X_c$	− 0·1
6996·31 (6)	14289·3	$\gamma\,3, 4\ R_c$	+ 1·2
7005·55 (3)	14270·5	$\gamma\,3, 4\ X_c$	− 1·9
7033·16 (5)	14214·4	$\gamma\,4, 5\ R_c$	+ 1·7
7043·13 (4)	14194·3	$\gamma\,0, 2\ R_a$	− 0·7
7071·28 (4)	14137·8	$\gamma\,5, 6\ R_c$	+ 0·4
7079·55 (3)	14121·3		
7109·83 (2)	14061·2	{ $\gamma\,6, 7\ R_c$ $\gamma\,2, 4\ R_a$	− 0·7 + 0·5
7127·18 (2)	14026·9		
7335·62 (2)	13628·4		
7426·31 (2)	13461·9	$\gamma\,4, 6\ R_b$	+ 4·2
7468·24 (3)	13386·4		
7484·41 (3)	13357·4	$\gamma\,3, 5\ R_c$	+ 1·4
7541·49 (4)	13256·3		
7597·51 (2)	13158·6	$\gamma\,2, 5\ R_a$	+ 1·1

Table 2. *R* heads of bands in violet-blue system α

v'' → / v' ↓		0	Δ''a Δ''b Δ''c	1	Δ''a Δ''b Δ''c	2	Δ''a Δ''b Δ''c	3	Δ''a Δ''b Δ''c	4
0	a	21639·8 83·8	931·3	20708·5 83·7		......		18868·9 84·3	910·6	17958·3
	b	21556·0 12·9	931·2	20624·8 12·6		......		18784·6	908·6	17876·0
	c	21543·1	930·9	20612·2		......		......		17863·4
	Δ'a	814·7		815·5						
	Δ'b	812·6								
	Δ'c	814·3								
1	a	22454·5 85·9	930·5	21524·0		......		......		
	b	22368·6 11·2		......		......		......		
	c	22357·4		......		20513·3 10·9	914·8	19587·6		
	Δ'a	808·9								
	Δ'b									
	Δ'c							804·6		
2	a	......								
	b	23177·5	930·8	22246·7 12·1		......		20403·5		
	c	......		22234·6		......		20392·2		
	Δ'a									
	Δ'b									
	Δ'c			799·5				802·4		
3	a			......		......		......		
	b			......		22121·3		......		
	c			23034·1	922·6	22111·5 9·8	916·7	21194·6		
	Δ'a									
	Δ'b					797·8				
	Δ'c					794·9				
4	a					......				
	b					22919·1				
	c					22906·4				

Table 4. Unidentified heads (*X*) of bands in yellow system β

v'' → / v' ↓		0	Δ''a Δ''b Δ''c	1	Δ''a Δ''b Δ''c	2	Δ''a Δ''b Δ''c	3	Δ''a Δ''b Δ''c	4
0	a	17993·3 248·6								
	b	17744·7 279·4								
	c	17465·3								
	Δ'a	839·5								
	Δ'b	839·2								
	Δ'c	839·0								
1	a	18832·8 248·9	931·2	17901·6 248·2						
	b	18583·9 279·6	930·5	17653·4 278·4						
	c	18304·3	929·3	17375·0						
	Δ'a			832·1						
	Δ'b			834·0						
	Δ'c									
2	a			......		17812·1 249·5	918·2	16893·9		
	b			18485·5 276·5	922·9	17562·6 277·8	....	......		
	c			18209·0	924·2	17284·8	....	......		
	Δ'a					824·4				
	Δ'b					824·9				
	Δ'c					827·6				
3	a					18636·5 249·0	918·8	......		
	b					18387·5 275·1	....	17193·7		
	c					18112·4	....	....		
	Δ'a									
	Δ'b									
	Δ'c							....		
4	a							18537·6 245·9	906·9	17629·7
	b							18291·7	....	......
	c							......		
	Δ'a									....
	Δ'b									....
	Δ'c									808·4
5	a									18438·1
	b									......
	c									......

Table 3. *R* heads of bands in yellow system β

v'' / v'		0	Δ″a Δ″b Δ″c	1	Δ″a Δ″b Δ″c	2	Δ″a Δ″b Δ″c	3	Δ″a Δ″b Δ″c	4	Δ″a Δ″b Δ″c	5	Δ″a Δ″b Δ″c	6
0	a	18007·4 247·2	930·7	17076·7 243·7	922·6	16154·1								
	b	17760·2 276·7	927·2	16833·0 278·9		15630·4								
	c	17483·5	929·4	16554·1	9237	······								
	Δ′a	838·8		839·2		······								
	Δ′b	836·9		835·8		······								
	Δ′c	838·2		838·0		839·5								
1	a	18846·2 249·1	930·3	17915·9 247·1		16409·9			······ 15837·3					
	b	18597·1 275·4	928·3	17668·8 276·7		······			······					
	c	18321·7	929·6	17392·1 276·7	922·2	······			······					
	Δ′a	······		831·2		830·9		826·3						
	Δ′b	······		829·7		······								
	Δ′c	······		832·3		······								
2	a	······		18747·1 248·6	923·1	17824·0 246·6	912·4	16911·6 248·0						
	b	······		18498·5 274·1	921·1	17577·4 276·6	913·8	16663·6 274·3						
	c	······		18224·4	923·6	17300·8	916·5	16389·3 274·3						
	Δ′a			825·1		823·5								
	Δ′b			822·9		824·4								
	Δ′c			825·5		820·1								
3	a	20502·4		······		18649·1 248·8	914·0	17735·1 247·1	904·4	······ 16583·6 279·0				
	b	······				18400·3 274·0	913·3	17488·0 276·6	904·8	16304·6	279·0			
	c					18126·3	916·9	17209·4 278·6						
	Δ′a					815·2								
	Δ′b					814·5								
	Δ′c					819·4								
4	a							18550·3 247·8	908·5	17641·8	811·6			
	b							18302·5 273·7						
	c							18028·8	909·6	17119·2				
	Δ′a							810·7						
5	a									18204·4	896·3	17308·1		
	b									17929·9 274·5		804·3		
	c													
6	a											18112·4		
7	a													{ ······ 17735·1 }

Table 5. R heads of bands in red-infra-red system γ

v'' / v'		0	$\Delta''a$ $\Delta''b$ $\Delta''c$	1	$\Delta''a$ $\Delta''b$ $\Delta''c$	2	$\Delta''a$ $\Delta''b$ $\Delta''c$	3	$\Delta''a$ $\Delta''b$ $\Delta''c$	4	$\Delta''a$ $\Delta''b$ $\Delta''c$	5	$\Delta''a$ $\Delta''b$ $\Delta''c$	6	$\Delta''a$ $\Delta''b$ $\Delta''c$	7
0	a	16048·4 292·1 313·4	930·0	15III74 290·2 312·7	923·1	14194·3										
	b	15756·3	929·1	14827·2		......											
	c	15442·9	928·4	14514·5		849·0											
	$\Delta'a$	849·0		8...·.													
	$\Delta'b$	847·9		846·6													
	$\Delta'c$	846·6		846·6													
1	a	16897·4 294·2 313·7	929·6	15967·8 294·0 312·7	923·6	15044·2 302·0 312·2										
	b	16603·2	929·4	15673·8	922·5	14751·3 312·2											
	c	16289·5	928·4	15361·1	922·0	14430·1											
	$\Delta'a$	8...·8		8...·8		842·4											
	$\Delta'b$	847·9		840·1		840·5											
	$\Delta'c$					840·3											
2	a	17741·7	931·1	16810·6 296·7 314·4	924·0	15886·6 295·8 311·4	916·2	14970·4 295·6 311·7	909·2	14061·2	902·6	13158·6				
	b	......		16513·0	923·1	15590·8	916·0	14674·8		......		......					
	c			16202·5	923·1	15279·4 311·4	9153	14364·1 311·7									
	$\Delta'a$			835·1		837·7		835·9									
	$\Delta'b$					833·5											
	$\Delta'c$					834·0											
3	a			8344	921·4	16724·3 300·0 310·0	916·7	15807·6 299·6 310·5	910·5	14897·1 300·3 307·5	902·9	......				
	b			17645·7		16424·3	916·3	15568·0	911·2	14596·8		13386·4					
	c			17036·9	922·6	16114·3 310·0	916·8	15197·5 310·5	908·2	14289·3 307·5							
	$\Delta'a$			826·5				830·9									
	$\Delta'b$							828·8									
	$\Delta'c$							826·5									
4	a			......		......		16638·5 303·4 311·1	910·3	15728·2 302·0 308·8	901·0	14827·2 306·4	892·4	......		
	b			......		......		16335·1	908·9	15446·2	905·4	14520·8 306·4		13628·4			
	c			17863·4		......		16024·0 311·1	906·6	15117·4 308·8	903·0	14214·4					
	$\Delta'a$							826·5		825·9		823·8					
	$\Delta'b$									823·2		820·1					
	$\Delta'c$									820·1							
5	a					18379·2		......		16554·1	903·7	15650·4 305·8	899·1	14751·3	813·4		
	b					......		......		......		15344·6 305·1	899·7	......			
	c					17760·2		......		15937·0		15034·5 310·1		14137·8			
	$\Delta'a$									819·6							
6	a											......	895·6	813·4	890·0	......	890·0
	b											......		15263·6 312·4		......	
	c											15846·8		14951·2 312·4		14061·2	
	$\Delta'a$											812·3					

Table 6. Unidentified heads (X) of bands in red-infra-red system γ

$v''\backslash v'$		0	$\Delta''a$ $\Delta''b$ $\Delta''c$	1	$\Delta''a$ $\Delta''b$ $\Delta''c$	2	$\Delta''a$ $\Delta''b$ $\Delta''c$	3	$\Delta''a$ $\Delta''b$ $\Delta''c$	4	5
0	a	$\Big\{$16033·8 293·1	931·4	$\Big\{$15102·4 295·0		$\Big\{$.......					
	b	15740·7 314·5	933·3	14807·4 314·0		14732·5 310·3					
	c	15446·2	932·8	14493·4	925·9	14422·2					
	$\Delta'a$ $\Delta'b$ $\Delta'c$	849·4		851·0	922·4						
				851·0							
				851·2							
1	a	$\Big\{$.......	931·7	$\Big\{$15953·4 295·0		$\Big\{$841·3					
	b	16590·1		15658·4 313·8	925·9	844·4					
	c	....		15344·6	922·4	$\Big\{$15871·2 297·4	917·1				
	$\Delta'a$ $\Delta'b$ $\Delta'c$			843·4		15573·8 310·2	918·1				
				843·?		15263·6					
				844·8		837·4					
						837·9					
						834·5					
2	a	$\Big\{$.......		$\Big\{$16796·8 208·0	925·6	$\Big\{$16708·6 206·0		$\Big\{$.......			
	b	....		16498·8 309·4	925·0	16411·7 313·6		14656·7 311·2	908·8		
	c	....		16189·4	925·8	16098·1		14345·5	9??·1		
	$\Delta'a$ $\Delta'b$ $\Delta'c$							836·9	911·5		
								836·5			
3	a	$\Big\{$.......		$\Big\{$.......		$\Big\{$15790·1 206·4	918·5	836·9		$\Big\{$14881·3 207·0	
	b	17958·3		....		15191·6 314·0	9?8·?	836·5		1488·1	
	c			....		15182·0 314·0	916·1	$\Big\{$15790·1 206·4		14270·5 313·8	
	$\Delta'a$ $\Delta'b$ $\Delta'c$					837·4		15182·0 314·0		831·4	
						837·9		831·9			
						834·5		827·5			
4	a							$\Big\{$16622·0	909·3	831·9	
	b							16121·1		15712·7	
	c							....		15102·4	
	$\Delta'a$ $\Delta'b$ $\Delta'c$										
5	a										$\Big\{$15635·2 ....
	b										....
	c										

LATTICE-DISTORTION OF COLD-DRAWN CONSTANTAN WIRE

By W. A. WOOD, M.Sc., Physics Department,
National Physical Laboratory

ABSTRACT. The space-lattice of constantan is found to undergo large distortion as a result of cold-working. A method is described for determining the variation in distortion quantitatively as constantan wire is cold-drawn, and the variation across the section of the wire during the drawing. It is found that (*a*) the distortion on drawing increases quickly to a steady maximum, which is maintained despite further drawing; (*b*) orientation does not begin until the maximum distortion appears; (*c*) just below the surface in the less drawn wires is a region of diminished distortion, but as the wire is further drawn the degree of distortion evens out across the section (this is explained in terms of the action of the die on the surface); (*d*) the temperature-coefficient of electrical resistance exhibits changes in drawing similar to the variation in distortion, and the two properties are practically proportional.

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§ 1. INTRODUCTION

THE space-lattice of constantan was found to be very susceptible to distortion as a result of cold-working. The chief object of the present work was to determine (i) the change in degree of distortion of constantan wire as its diameter was gradually reduced by cold-drawing, (ii) the change in degree of distortion across the section of a wire at a given stage of the drawing, and (iii) the relation, if any, between the degree of distortion thus produced and the variation in the value of the temperature-coefficient of electrical resistance which results when constantan wire is hard-drawn[1]. The point where crystal orientation is introduced by the drawing was also sought.

The term "lattice-distortion of constantan" is used herein solely in the sense of a permanent, irregular displacement of atoms from their normal positions at the face-centres and corners of the cubic unit cells which form the basis of the constantan lattice. A number of the cells therefore cease to be perfect cubes. This type of distortion is recognized by its influence in broadening the lines of the X-ray diffraction spectrum of the distorted material[2, 3]. It differs from "distortion by slip," where the grains of a worked metal are deformed by movement along slip planes, inasmuch as this process leaves the breadth of the diffraction lines unaltered[2]. On this account distinction between the two types could be made by experiment. It differs also from the change in size of a unit cell, sometimes referred to as distortion[4], which occurs when some of the atoms of a metal are replaced by atoms of another in order to form an alloy of the solid-solution type.

This change in size does not affect the breadth of the diffraction lines and is presumably the same therefore, on the whole, for every cell. In the last two cases no irregularity is introduced into the lattice.

§ 2. EXPERIMENTAL PROCEDURE

It was necessary first to secure an appropriate range of specimens, and secondly, to select a method of estimating quantitatively the degree of distortion.

Two sets of constantan wire were finally used. The first was obtained by cold-drawing a wire, previously annealed and thus rid of distortion, as shown by its X-ray spectrum, from an initial diameter of 1·645 mm. by twenty-eight steps to 0·726 mm. At each step of the drawing a length was severed from the parent wire and its X-ray photograph secured. The second set (kindly supplied by the Research Department of Metropolitan Vickers-Electrical Co., Ltd.) was obtained from a wire of diameter 0·762 mm. drawn by twelve steps to 0·102 mm. Measurements of the temperature-coefficient of electrical resistance of these specimens between 0 and 40° C. were made in the Electrotechnics Division of the National Physical Laboratory. By measuring the distortion from the X-ray photographs of the severed specimens the change in degree of distortion with the percentage reduction of diameter of the initial wire could thus be determined.

The variation of distortion across the section of a wire was obtained by taking an X-ray photograph of the surface of the specimen, then etching down the wire slightly by electrolytical dissolution in dilute hydrochloric acid [5], and finally re-photographing the new surface. By repetition of the process of etching and photographing, the degree of distortion was determined from the outside to the core of typical wires.

The estimation of distortion was based on the change in breadth of the (331) line of the X-ray spectrum. This line under the conditions of experiment was a doublet. The separation of the components due respectively to the α_1 and α_2 constituents of the analyzing copper $K\alpha$ radiation, was of the order of 1 mm. For any wire the breadth of the line in a Debye photograph will depend on its diameter, its absorption, the radius of the camera, the divergence of the incident beam, and lastly, the degree of lattice-distortion in the wire. These complications were allowed for experimentally by the following procedure. Consider the specimens drawn from the wire of diameter 1·645 mm. First the breadth of the (331) line in the photographs of these specimens was measured, as described below, with the aid of a Moll microphotometer. This breadth was plotted against the known percentage reduction of diameter of each specimen. Next, a number of the specimens were annealed until free of distortion, and then photographed under exactly the same conditions as before. The wires were annealed for 1 hour at about 650 C. in an electric furnace through which was circulating purified argon. This atmosphere was chosen to prevent change in diameter of the wires as a result of oxidation. The measurements of the breadths of the (331) line were now repeated and again plotted, on the same diagram as before, against the percentage reduction of

diameter. Now the only factor distinguishing this curve from the first is that of lattice-distortion, which is present in the first only. The other factors have remained unchanged. Therefore, for a given value of the abscissae (the percentage reduction of diameter) the difference in height of the ordinates (line-breadth) gives the change in breadth of the (331) line caused solely by distortion. Finally, the second curve was subtracted in this way from the first and a resultant third curve was obtained giving the change in breadth of the (331) line for each percentage reduction in diameter as the initial wire was drawn down. This change in breadth was taken as a measure of the degree of lattice-distortion.

The process was repeated for the wire drawn down from the initial diameter of 0·762 mm. and also, with one modification, for the specimens obtained by etching down the wires. This modification was necessitated by the fact that the wire at any stage of the etching could not be annealed, since it was always required for further etching. It involved therefore the numerical deduction of the breadth of line which these specimens would give if they were annealed from the second curve described above. This curve shows the breadth of an undistorted line for a wire of known diameter in the range 1·645 mm. to 0·726 mm. The corresponding graph of the second set of wires gives a curve for the range 0·762 mm. to 0·051 mm. The diameter of the etched wires fell in one or other of these ranges.

Measurements of the (331) line were made as follows. The photometer records give the distribution of intensity across the width of the line. The line appears as two neighbouring peaks, approximately triangular, set on a line of continuous background. One peak corresponds to the α_1 and the other to the α_2 wave-length. On broadening, the two peaks coalesce. But since the intensity of the α_1 peak is known to be about twice that of the α_2, the point of maximum intensity of the compound line gives the position of the α_1 peak. The point of minimum intensity of the α_1 line is taken as the point where the trace of the peak descends to the line of continuous background on the side of the peak opposite to that complicated by the presence of the α_2 component. The distance between the first point and the second measured along the base line of continuous background gives a measurement of half the breadth of the α_1 line uncomplicated by the α_2 component. This is the value used in constructing the graphs. If B_1 is the value obtained for a distorted wire and B_2 the corresponding value after the distortion was removed by annealing, then $B_1 - B_2$ is the quantity taken as a measure of the distortion. This quantity is denoted by S.

B_1

B_2

S

The value of S as it stands is an arbitrary expression of length. It can, however, be compared with other work when the experimental data which are stated herewith are considered. The radius of the circular camera used was 5 cm.; the magnification factor introduced by the Moll microphotometer is such that 1 cm. on the film corresponds to 7 cm. on the photometer record; the value of the reflection angle θ for the (331) line was 70·7°. Further interpretation of S can be made only when the exact position of the atoms after distortion is known. This is not known at present.

If we assume, however, the change in breadth of the line to follow from the

variation of spacing about the normal value, and that the increase in breadth on distortion corresponds to the maximum change of lattice parameter, then S may be related to the fractional change of spacing of the (311) planes as follows. From the Bragg relation $2d \sin \theta = \lambda$, we have by differentiation

$$\delta d/d = - \cot \theta \, \delta\theta,$$

d, θ, λ where d is the spacing of the planes, and θ the angle at which the wave-length λ is reflected by those planes. Now S measured in mm. on the photometer record corresponds to a distance S 7 mm. on the photograph, and this distance subtends an angle $\delta\theta = S/350$ at the centre of the camera, the radius of the camera being 50 mm. This angle is the angle subtended by the change in the breadth of the line in the direction of increased spacing, so that for the maximum fractional increase of spacing

$$\delta d/d = (S/350) \cot \theta$$
$$= 0 \cdot 001 \, S, \text{ since } \cot \theta = 0 \cdot 3502,$$

and the percentage increase in spacing is $0 \cdot 1 \, S$, where S is measured in mm. This value may be taken as giving the order of magnitude of the distortion.

The photographs were also watched for the appearance of crystal-orientation as shown by irregular distribution of intensity along the spectral lines. Orientation first appeared at the core of the wire, as was shown by the etched specimens. The diameter of the wire when orientation at the core was first found was noted for each set of drawn wires.

§ 3. RESULTS

Orientation was observed to begin at the core in the first set of wires drawn down from the initial diameter of 1·645 mm. when the reduction was 42 per cent. In the second set of wires drawn from the diameter of 0·762 mm. the presence of orientation was first noted at the reduction of 61 per cent. Some difference between the two sets was to be expected, since the wires obtained from different sources were drawn by different stages. (There appears to be room for research on the effect of different speeds of drawing on the rate of production of distortion and orientation.)

The results of the distortion measurements are recorded on the accompanying graphs. Figure 1 shows, in the upper curve, the way in which the half-breadth B_1 of the (331) line, as measured on the photometer records, varies as the diameter of the wire was drawn from 1·645 mm. The influence of the distortion is striking. The lower curve gives the variation of the half-breadth B_2 of the (331) line for the corresponding undistorted specimens. This is virtually a curve which calibrates the circular camera by giving the breadth of the line as a function of the diameter of normal wire specimens photographed at the centre of the camera under the particular experimental conditions employed.

Figure 2 is obtained by subtracting the second curve from the first in figure 1,

in the manner already described. The resultant value S, the actual broadening due to distortion, and the quantitative measure thereof, is plotted against the percentage reduction of diameter on the same scale as figure 1. On the right side of the graph the ordinates are expressed in terms of the equivalent percentage change in size of the (331) spacings. The first point of interest is that the curve is made up of two
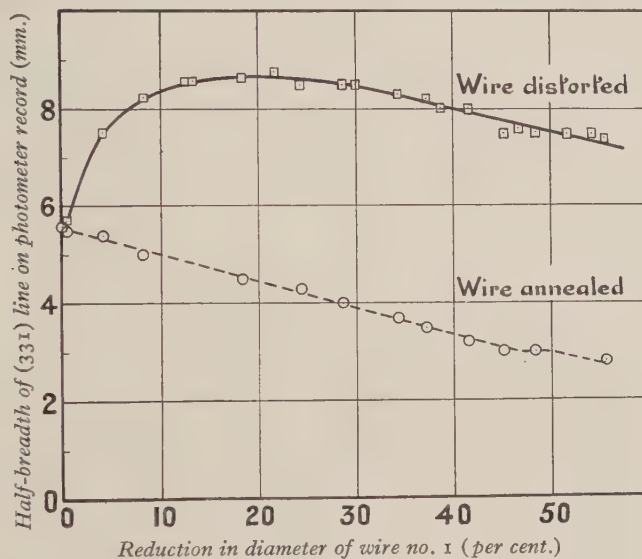
Fig. 1. Effect of drawing on half-breadths B_1 and B_2. (Initial diameter of wire 1·645 mm.)
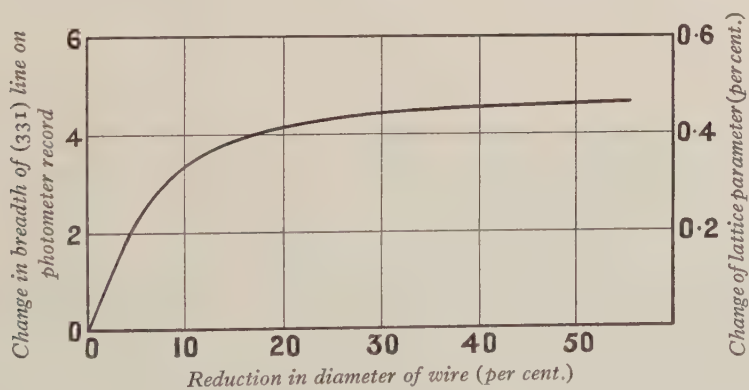
Fig. 2. Effect of drawing on distortion S. (Initial diameter of wire 1·645 mm.)

parts, (i) the quick rise in degree of lattice-distortion in the early stages of drawing, and (ii) the region of a maximum value to which the degree of distortion steadies up as the wire is further drawn. The second point of interest is that orientation only begins to make its appearance when the distortion has practically attained the steady maximum value.

Figure 3 shows the variation in degree of distortion across the section of three typical specimens at which the external diameter had become 1·578 mm., 1·500 mm.

W. A. Wood

and 0·852 mm. respectively. The value of S for each specimen is plotted against the decrease in diameter as the surface was etched away. It appears that in the less drawn wires there is a region of diminished distortion just under the surface. This diminution disappears, however, on further drawing, and in the later stages the distortion is practically constant across the wire. This observation agrees with that of Burgers[3] on the sharpness of the Kz_1, z_2 doublet reflected at different depths within tungsten single-crystal and polycrystalline wire. The explanation advanced here is that, at first, the distortion due to drawing is greatest at the core of the wire and decreases towards the surface. At the surface additional distortion is caused by the mechanical action of the die[5] and this distortion will decrease toward the centre of the wire. The superposition of the two effects will produce
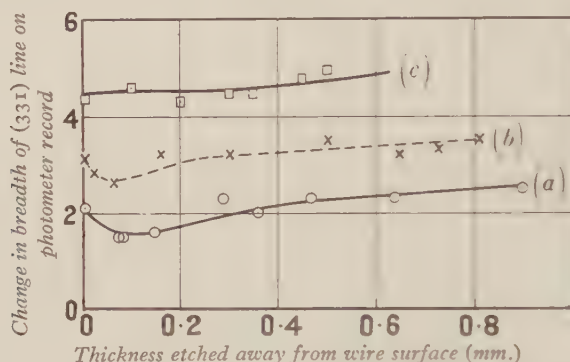
Fig. 3. Variation of S across section. Initial diameters: (a) 1·578 mm.; (b) 1·500 mm.; (c) 0·852 mm.

the diminution observed below the surface. As the wire is further drawn the steady maximum value of distortion will spread over the whole wire. It must be noted that at first, therefore, the mean distortion taken over the whole section of the wire will be somewhat less than that measured at the surface and recorded in figures 1 and 2, but that this discrepancy will diminish with drawing. The influence of this point is referred to in connection with figure 4 (b).

Figure 4 (a) deals with the second set of wires drawn from the initial diameter of 0·762 mm. It gives the temperature-coefficient of electrical resistance of the wires plotted against the percentage reduction of diameter. Figure 4 (b) gives the change in degree of distortion of the same wires plotted against the percentage reduction of diameter on the same scale as figure 4 (a). The first observation of note is that in each case we have a similar type of curve. The maximum of the distortion is, however, reached before that of the temperature-coefficient. This is to be expected since, as discussed in connection with figure 3, the mean value of the distortion throughout the wire is less at first than the distortion at the surface recorded in the graphs. It is the mean value which should be taken into account in the comparison of figure 4 (b) with the temperature-coefficient curve. This consideration would increase the resemblance. Unfortunately the application of an

accurate numerical correction based on figure 3 is complicated by the difficulty of maintaining a constant diameter during the etching. We emphasize, however, the similarity in the early rise and the attainment of a steady maximum at virtually the same reduction. The second point of note is that the orientation again does not occur until the distortion has reached its maximum value. Further drawing therefore, whilst increasing the amount of orientation, leaves the temperature-coefficient of resistance unaltered. This contradicts the view, sometimes advanced[6], that orientation affects electrical properties in metals based on a cubic lattice.
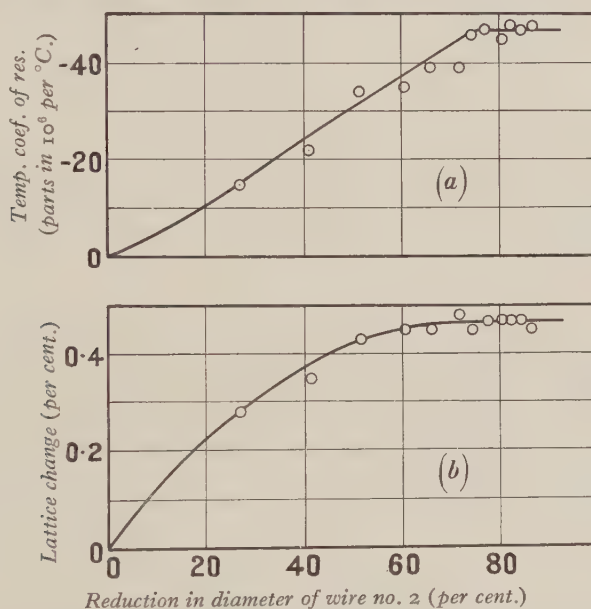
Fig. 4. Lattice change and temperature-coefficient of resistance.
(Initial diameter of wire 0·762 mm.)

§ 4. DISCUSSION OF RESULTS

The above results give rise to the following considerations.

(i) Orientation begins when distortion in the constantan is a maximum. This suggests that the process of deformation produces, in order, lattice-distortion, slip, and orientation as a result of slip. The range occupied by distortion may be negligible as in pure aluminium[2], small as in pure copper, or large as in constantan and steel.

(ii) The cold-drawing of constantan produces a maximum value of lattice-distortion which further drawing merely maintains. It is suggested that the maximum distortion attainable is a definite quantity for one metal or alloy, and different for different metals or alloys. It is hoped to follow up this point.

(iii) The variation of the temperature-coefficient of resistance of constantan is practically proportional to the change in degree of distortion produced by cold-work. It would follow that a material capable of exhibiting a large stable change in

lattice-distortion will also be capable, as a result of some treatment, of possessing a wide range in the value of given physical properties influenced by the lattice-disturbance. Thus the effect of alloying iron with other elements may well be merely one means of varying the range of distortion which the iron lattice will withstand, and consequently the range of certain properties—a point so characteristic of steel. On the other hand, a metal which orients quickly on being rolled or drawn would have fairly constant properties as far as the distortion factor is concerned, whatever the heat or work treatment.

The degree of distortion a lattice will exhibit before it slips or breaks down appears to be a quantity of fundamental physical importance.

§5. ACKNOWLEDGMENTS

In conclusion the author wishes to record his thanks to Dr G. Shearer for help in securing specimens, and to Dr G. W. C. Kaye for his interest and for providing facilities necessary for the research.

REFERENCES

(1) R. S. J. SPILLSBURY, *J. Sci. Inst.* **6**, 357 (1929).
(2) V. DEHLINGER, *Z. für Krist.* **65**, 615 (1927).
(3) W. G. BURGERS, *Z. für Phys.* **58**, 11 (1929).
(4) W. L. FINK and K. R. VAN HORN, *Inst. of Met.* **44**, 241 (1930). (Discussion.)
(5) W. A. WOOD, *Phil. Mag.* **11**, *Suppl.* 610 (1931).
(6) R. W. DRIER and C. T. EDDY, *Trans. Am. Inst. Met. Eng., Inst. Met. Sec.* p. 140 (1930).

DISCUSSION

Dr G. SHEARER referred to the fact that selective orientation begins to make its appearance only when distortion has practically attained its maximum value (p. 71). It would be interesting to find out whether a similar rule holds in the case of other alloys and metals, and if so, whether the point where distortion is a maximum and selective orientation begins is masked by changes in other physical properties.

Dr E. H. RAYNER: Methods of studying the internal structure and stability of nickel-copper alloys are of special interest from the point of view of their use for standard resistances. The reason is that a very small temperature-coefficient is desired in such resistances in order to be able to attain accuracy of measurement of the order of a few parts in a million. Most pure metals have coefficients of the order of 4000 parts per million per degree centigrade, but it happens that if nickel be added to copper to the extent of Cu 66 to Ni 34, the coefficient becomes zero and thereafter somewhat negative, until at a ratio of Cu 45 to Ni 55 it is zero again, rising at 100 per cent. Ni to a value of 6700, that for copper being 4300. Constantan is the term given to the alloy having zero coefficient with the larger proportion of copper. It is more ductile and cheaper than the other.

The disadvantage of the copper-nickel alloy is its large thermo-e.m.f. against copper, about $40\,\mu$V./$^\circ$C. Weston, of America, found that the addition of a suitable proportion of manganese would reduce this to about $2\,\mu$V., while retaining a low temperature-coefficient. Secular stability of resistance has, however, proved difficult to attain, and any information on the internal structure of manganin and other alloys suitable for use in the making of standard resistances cannot fail to be of value.

Mr B. P. DUDDING: There is one aspect of this paper to which I would like to call attention. The author claims to have established that distortion of the lattice takes place by cold-working. In similar studies made on tungsten it was never possible to say definitely that the diffusion in the ray pattern was produced by lattice-distortion, since the presence of very small crystallites produced by cold-work would cause similar diffusion. If we assume that the author can establish his claim to differentiate between these two effects, then it would be of interest to know if the anomalous behaviour of nickel-chrome alloys with regard to change of resistivity, on being cold-worked, can be shown to be associated with a similar lattice-distortion.

AUTHOR'S reply: Work on the lines suggested by Dr Shearer is in progress, and the relation between onset of orientation and distortion appears to hold for a number of other metals.

Mr Dudding wonders whether it is possible to distinguish the diffusion caused by distortion from that caused by the presence of very small crystallites. This is merely a matter of accuracy of measurement, and when the broadening is large, as for constantan, then there should be no difficulty in differentiating between the two effects. For, firstly, the change in breadth of a line depends on its angle of diffraction in a way which is different in the two cases. Also, secondly, the presence of distortion may, and usually does, produce a shift in the position of maximum intensity of the spectral lines; such a shift cannot occur in the case of the small-crystal effect. This aspect of the question has been treated in many recent researches such as those referred to in the text, and, on that account, was not given prominence in the paper.

WIRELESS ECHOES OF SHORT DELAY

By E. V. APPLETON, F.R.S. and G. BUILDER, B.Sc.

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ABSTRACT. An account of a simple method of producing short pulses of radio-frequency energy is given, together with notes on its application in the investigation of wireless echoes of short delay. Details of simultaneous visual and photographic methods of delineating such echoes are also described. The discussion of sample records and results serves as a basis for drawing conclusions concerning the relative advantages of the frequency-change and group-retardation methods of investigating the ionized regions of the upper atmosphere.

§ 1. INTRODUCTION

ONE of the two methods most commonly used for exploring the electrical structure of the upper atmosphere is that involving the measurement of the time required for a brief wireless signal to travel upwards to the ionized region and back. This quantity is most conveniently determined by arranging for an emitting station to send out very short pulses of radio-frequency energy and measuring, at a point a short distance away, the difference between the times of arrival of a particular signal pulse via the ground and via the atmosphere. It is

U readily seen that such measurements give the quantity $\int \frac{ds}{U}$ for the atmospheric wave track, where U is the group velocity with which the waves travel along an element of path ds. Moreover, since, for the portion of the wave track within the

c ionized region, U is less than the velocity c of electromagnetic radiation in free space, the equivalent path, $c \int \frac{ds}{U}$, of the atmospheric waves is greater than the actual

s path s. The equivalent height of the reflecting region, obtained by simple triangulation from the atmospheric wave equivalent path and the ground path, is therefore greater than the actual height at which deviation takes place.

In a previous communication[*] a comparison was made of the two methods principally used for equivalent-height measurement, and it was shown that both the frequency-change method and the group-retardation method (that mentioned above) give substantially the same result. Up to the present, however, the frequency-change method has been used predominantly in England while the group-retardation method has been similarly used in America. Although, as has been mentioned, the two methods should give similar results, somewhat divergent conclusions have been drawn by the British and American investigators from their observations. It therefore seemed to us desirable to develop the use of the group-retardation method in this country and compare it with the frequency-change method we have previously used exclusively.

[*] E. V. Appleton, *Proc. Phys. Soc.* **41**, 43 (1928).

The present communication gives a brief account of a simple method of producing the short pulses required in the group-retardation experiment together with notes on possible methods of delineating the short-delay echoes which occur. Some sample records are also discussed and serve as a basis for drawing conclusions as to the relative advantages of the frequency-change and group-retardation methods of measuring equivalent heights. It is hoped to give a more detailed account of observations made by the echo method in a future communication.

§ 2. THE PRODUCTION OF SHORT RADIO-FREQUENCY PULSES

Various methods of producing the short radio-frequency pulses necessary for the group-retardation experiment have been previously described. Breit and Tuve* and Dahl and Gebhard† used an alternating voltage of $500\sim$, amplified by a transformer, and applied to the grid of a set of amplifying tubes which linked a master oscillator to the oscillator proper. By applying also simultaneously a fixed negative grid-bias to the amplifying tubes it was arranged that these tubes functioned during part only of each positive half-cycle of the grid potential. In this way radio-frequency pulses having a duration of about 0·0007 sec. spaced 0·002 sec. apart were obtained. As a result of our own experiments we believe that such pulses, because of their relatively long duration and close spacing in time, could hardly have been very satisfactory.

Later Tuve and Dahl‡ obtained shorter pulses by using a multivibrator circuit in an unbalanced condition. A transformer in the plate-supply circuit of the multivibrator translated the square-topped current pulsations into voltage pulses, alternately positive and negative, of very short duration. These voltage pulses were applied to the grids of amplifying tubes already strongly negatively biassed so that only positive pulses were effective. Pulses of 0·00025 sec. duration were obtained in this way.

More recently Goubau§ and Goubau and Zenneck‖ have described experiments in which the low-frequency pulses applied to the grids of the amplifying tubes were produced by means of a highly-saturated iron-cored choke, connected in series with a condenser and the secondary of a transformer, the primary of which was fed by a 500-cycle generator. Pulses of duration 0·0001 sec. were obtained¶.

In the experiments to be described no special modulating device has been used at all. It has been found that if the grid leak of an ordinary continuous-wave triode transmitter is increased to a high value, the generator automatically produces the desired short pulses of radio-frequency energy alternating with uniform periods of quiescence. This property of a triode oscillator has been previously used in pro-

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* *Phys. Rev.* **28**, 554 (1926).   † *Proc. Inst. Rad. Eng.* **16**, 290 (1928).
‡ *Proc. Inst. Rad. Eng.* **16**, 794 (1928).   § *Phys. Zeit.* **31**, 333 (1930).
‖ *Zeit. für Hochfrequenztech.* **37**, 207 (1931).
¶ Since the above was written E. L. C. White has published (*Proc. Camb. Phil. Soc.* **27**, 445 (1931)) an account of two novel methods of producing the radio-frequency pulses. Short flashes of light of duration 0·00025 sec. were obtained by mechanical means and were used to modulate a wireless transmitter via a photo-electric cell and amplifier.

viding a linear and unidirectional time-base for cathode-ray oscillographic de-lineation of wave-form, and an account of its action has been already given*. Briefly the action depends on there being a difference between the mean negative grid potentials for starting and stopping the generation of oscillations. When oscillations start, the partial insulation of the grid causes the mean grid potential rapidly to assume a negative value numerically greater than that required for stopping the oscillations. The oscillations are thereby rapidly quenched and do not start again until the negative grid potential has fallen (by the leakage of electrons from grid to filament via the high-resistance leak) to the appropriate value for the re-starting of oscillations.

§3. THE EMITTING STATION

The sending station used in these experiments was located in the Electrical Engineering Department at East London College, Mile End Road, and consisted of a series-fed valve oscillator of tuned-grid tuned-anode type. This type of circuit was preferred because with it the excitation can be readily adjusted to any desired extent.
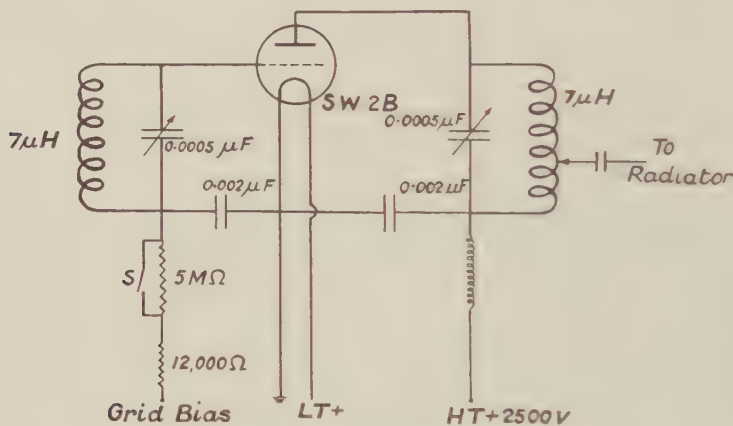
Fig. 1. Emitter circuit.

Figure 1 shows the circuit with the values of the components used. Small in-ductance/capacity ratios were used in the tuned circuits and, with only $1\,\mu$F across the anode supply and switch S closed, a pure steady c.w. signal was generated. A Mullard SW 2 B (200 watts anode dissipation) was employed, the requisite filament and plate voltages being obtained from a 500-watt Newton motor-generator yielding 2500 and 12·5 volts for the anode and filament supplies respectively.

The radiating system was a half-wave Hertz antenna suspended horizontally between poles standing about 20 ft. above the buildings. This was fed through a single wire from a tapping on the transmitter plate inductance, a d.c. stopping

* E. V. Appleton, R. A. W. Watt and J. F. Herd, *Proc. Roy. Soc. A*, **111**, 672 (1926).

condenser being inserted in the feeder. To carry out with the same system experiments using the frequency-change method a small variable condenser (not shown in diagram) was tapped across a few turns of the anode circuit inductance. Variations of the capacity of this condenser gave the required small variations of emitted frequency.

To convert the system into a pulse-generator, switch S is opened so that a high resistance is introduced into the grid circuit. It is found that, with values as shown, pulses of duration of 0·0001 sec. or less, spaced in time 0·02 sec. apart, are generated. It will readily be seen that the duration of the pulse produced depends on the time required for the grid condenser to charge up negatively so that small values of this condenser favour the production of very short pulses. The periods of quiescence between successive pulses is determined by the time-constant of the grid circuit and thus by both the value of the grid condenser and that of the grid leak.

To check the nature of the signal pulses emitted, a local monitor oscillograph circuit was used. This consisted of a coil and rectifier, loosely coupled to the transmitter, the output of which was applied to one pair of deflecting plates of a cathode-ray oscillograph. A linear time-base voltage was applied to the other pair of plates and synchronized with the frequency of occurrence of the pulses. With the monitor system used in this way it was found that the optimum adjustment of the excitation was such that the mean anode current was a minimum. Such conditions gave the shortest pulses to be obtained with given grid-circuit constants. In fact this adjustment is really the only one necessary, and after we had learned this we found that a monitoring system was superfluous.

§4. THE RECEIVING STATION

The receiving apparatus was housed in the wireless hut on the roof of the East Wing of King's College, London, at approximately 5 km. from the sending station.

The receiving assembly was designed to permit measurements by both the frequency-change method and pulse-retardation method. It is shown schematically in figure 2. On account of the high "electrical noise-level" at King's College, especially in the day-time, a small frame was used almost exclusively as the antenna system. The radio-frequency amplifier was a two-stage screened-grid valve set of conventional design. This was followed by a balanced detector stage in which circuit an Einthoven galvanometer could be included for work with the frequency-change method.

For observations by the group-retardation method the signal output of the detector stage was fed into a low-frequency amplifier for magnification to a suitable level for recording with a high-speed oscillograph. Since the signals to be amplified consist, as has been mentioned, of very short pulses of about 10^{-4} sec. duration, it is necessary for the amplifier to have a uniform response over as wide a range of frequencies as possible. The detector and amplifier constants indicated in the diagram have been found suitable, but care must be taken, in assembling the set,

to minimize stray capacities so as to reduce high-frequency losses. Decoupling elements in the feed circuits were found useful in ensuring stability.

For photographic recording a Duddell three-element high-frequency oscillograph, made by the Cambridge Instrument Company, was used, the scale sensitivity of which is about 0·3 mm./mA. One of the vibrators was included in the anode circuit of a triode of high mutual conductance, the grid circuit of which was connected to the output stage of the low-frequency amplifier. To obtain a time-base on the record another vibrator was fed in a similar way from a low-frequency triode oscillator. The third vibrator was not used.
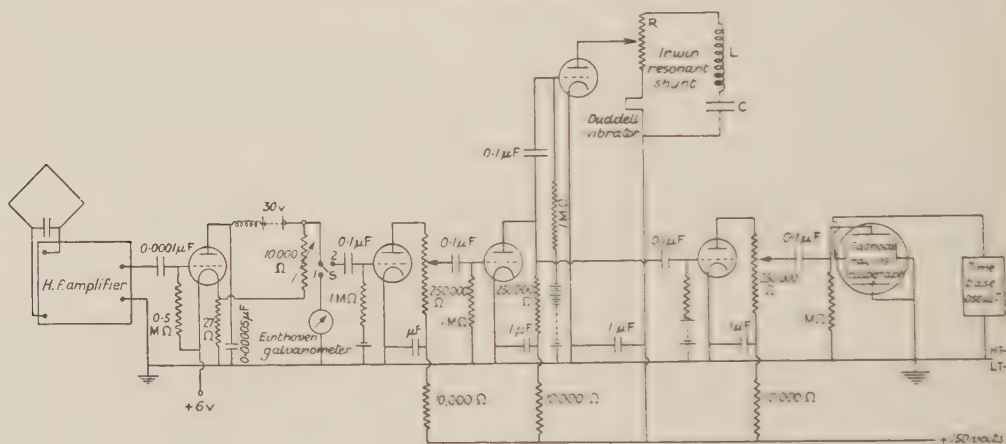
Fig. 2. Receiver circuits.

Much difficulty has been experienced owing to unsatisfactory damping of the oscillograph vibrators. The large changes in the viscosity of the damping oil, due to variations of temperature, make it impossible to maintain the moving systems in a critically damped condition. To obviate this difficulty the Irwin resonant-shunt method* of damping has been employed. In this method the electrical circuit, consisting of a series resonant shunt and the vibrator, is critically damped by means of the resistance R, figure 2, the most suitable tapping point on which depends on the natural damping of the vibrator. Although this method does not appear to be generally satisfactory with a bifilar suspension if the natural damping of the vibrating system is low, we have found that it works very well indeed with a vibrator about one-half damped in oil. In this connection it should be noted that the temperature changes of oil-viscosity affect not only the damping but also the natural frequency of the vibrator, so that a change of oil-temperature necessitates a slight adjustment of the shunt circuit, as well as an alteration of the tapping point on R, to restore optimum conditions.

For photographic recording of the received signals a rotating drum camera carrying a strip of sensitive paper about 20 in. long was used. The camera was

* J. T. Irwin, *Oscillographs*, p. 91.

belt-driven by a synchronous a.c. motor driven from the mains. Illumination was provided from a 60-volt 6-amp. arc with automatic feed.

For the visual observations a von Ardenne cathode-ray oscillograph, connected to the low-frequency amplifier as shown in figure 2, was used. The time-base, which was linear and unidirectional, was synchronized in stroke-frequency with the frequency of pulse-emission of the sending station, so that a stationary screen image was produced showing the ground pulse and any echo pulses present. A more detailed account of the circuit used for the production of the time-base voltage is given in an appendix. For visual work the oscillograph anode voltage was about 1500 volts, but this was increased to 3000 volts when it was desired to take photographs of the screen image.

§5. SOME TYPICAL RECORDS

In the plate, (*a*) to (*g*), are reproduced some typical records of the received pulses obtained with the Duddell oscillograph. They were all obtained at King's College with the sending station at East London College. The wave-length used throughout was 80 metres. Except where the contrary is stated the time scale shown under the record is that produced by an alternating current of 1115 $\sim$. In each case the first impulse (marked G) is that received direct via the ground, the subsequent pulses being due to waves reflected by the upper atmosphere.

Record (*a*) (0337 G.m.t. July 16, 1931) shows the reception of the ground signals G, G without any echoes. It is of interest in showing the sharpness of the pulses used.

Record (*b*) (0355 G.m.t. July 16, 1931) is to be compared with record (*a*) and shows the reception of a ground signal G followed by a singly-reflected signal F_1.

Records (*c*) (June 15, 1931) are of interest in confirming results previously obtained in England with the frequency-change method of measuring equivalent heights, in that they indicate reflections from two regions at different heights in the upper atmosphere. Record (*c*1) taken at 1830 G.m.t. illustrates a singly-reflected pulse E_1 from the lower of these two regions (region E). Record (*c*2) taken at 1850 G.m.t. shows a singly-reflected pulse E_1 from the lower region and a singly-reflected pulse F_1 from the upper region (region F). Record (*c*3) taken at 1910 G.m.t. shows that as sunset (2020 G.m.t.) was approached the singly-reflected pulse E_1 from the lower region was less intense while that from the upper region F_1 was much more marked. A pulse F_2 indicates double reflection from the upper region.

Record (*d*) (2300 G.m.t. June 18, 1931) illustrates a case of multiple reflection up to the fifth order. The first order echo F_1 shows signs of being composite.

Multiple reflection up to echoes of the eighth order has been observed at times.

Record (*e*) (2115 G.m.t. July 15, 1931) illustrates simultaneous singly-reflected signals from both regions E and F together with an echo S of comparatively long delay, the origin of which is doubtful since it cannot be related to the others in a simple manner.

6

Record (f) (0430 G.m.t. July 16, 1931) illustrates a phenomenon frequently observed during the night hours, namely the splitting of an echo from region F into two components F_1' and F_1''. It will be seen that in the second-order echoes only F_2' is appreciable.

Record (g) (2247 G.m.t. July 15, 1931) shows the splitting of both first and second-order echoes from region F.

Records (f) and (g) illustrate a phenomenon which was first observed some years ago using the frequency-change method. In this case the splitting of the down-coming waves into two components of slightly different equivalent path is shown as the superposition of two sets of interference fringes of slightly different period. For example, a typical early-morning record of such interference maxima and minima is shown in record (h) taken at 0330 G.m.t. on April 26, 1929, with a wave-length of 100 m. Here it will be seen that, in addition to the large number of fringes produced by interference with the ground waves and the down-coming waves, there are about $3\frac{1}{2}$ fringes due to interference between the two down-coming waves F_1' and F_1''. Record (g) is of interest also in showing the rapid variation in the intensity of the down-coming waves which very frequently takes place within a fraction of a second. Such variations of intensity are very strikingly shown on the cathode-ray oscillograph during visual observations.
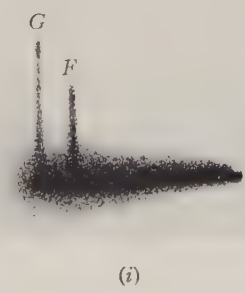
Record (i) is a snap photograph of the echo pattern commonly observed on the cathode-ray oscillograph screen. The ground signal G and a singly-reflected signal from region F are shown. The base-line frequency when this record was taken was 80 $\sim$, so that the whole length of the base corresponds to a time of about 12 milli-seconds.

§ 6. SOME EQUIVALENT-HEIGHT MEASUREMENTS

In figure 3 is shown the variation of equivalent heights obtained for the night July 15-16, 1931, using a wave-length of 80 metres and employing the group-retardation method.

It is seen that reflection took place at times from the Kennelly-Heaviside layer (region E) at a height of about 110 km., but that during the greater part of the night the waves penetrated to region F. It will further be seen that the reflected signals from region F were very frequently split into the two components F_1' and F_1''; examples of this occurrence have been given above. It is not definitely certain to what this splitting of the echo into two components is due; but a possibility is that we are dealing with the rays, elliptically polarized in opposite senses, which travel in the ionized region with different group-velocities, owing to the magnetic field of the earth.

The absence of echoes during the period from 0208 to 0335 G.m.t. is probably due to electron limitation, the 80-metre waves having penetrated both regions E and F. If this is so, since we are dealing with a case of approximately normal incidence, it is possible to obtain a superior limit to the ionization content of region F. This is found to be 3.5×10^5 electrons per cm.[3]

(a)

(b)

(c 1)   (c 2)   (c 3)   (d)

1110 c/s   1110 c/s   1110 c/s   1110 c/s

(e)

(f)

(g)

Frequency decreasing   Frequency increasing

(h)   (i)
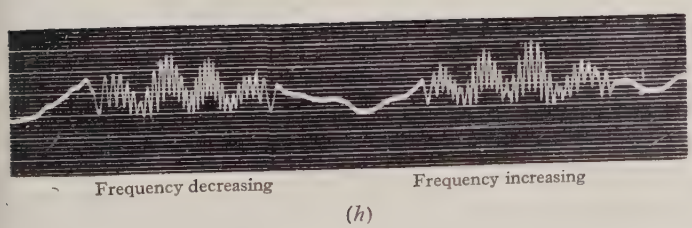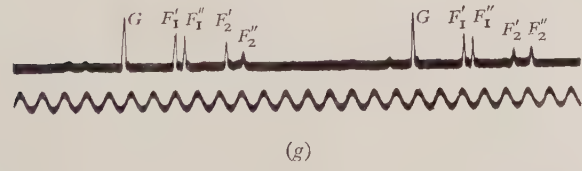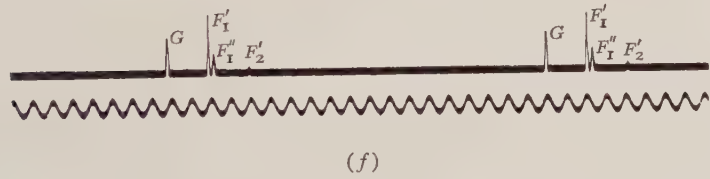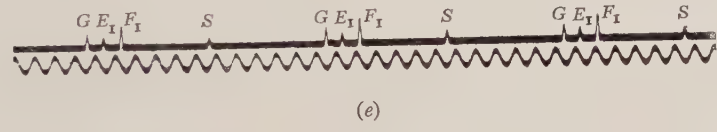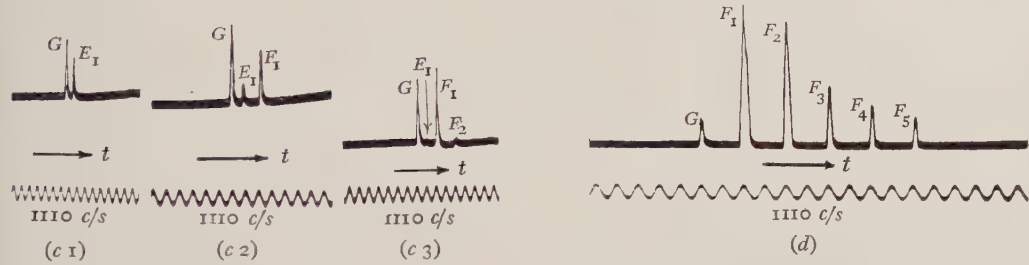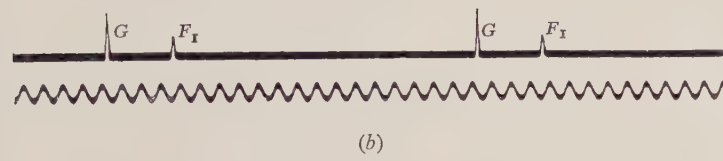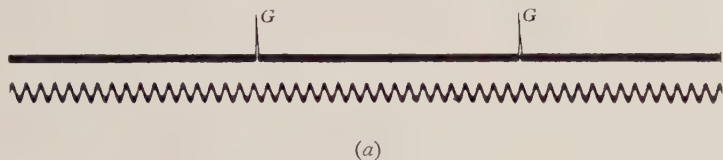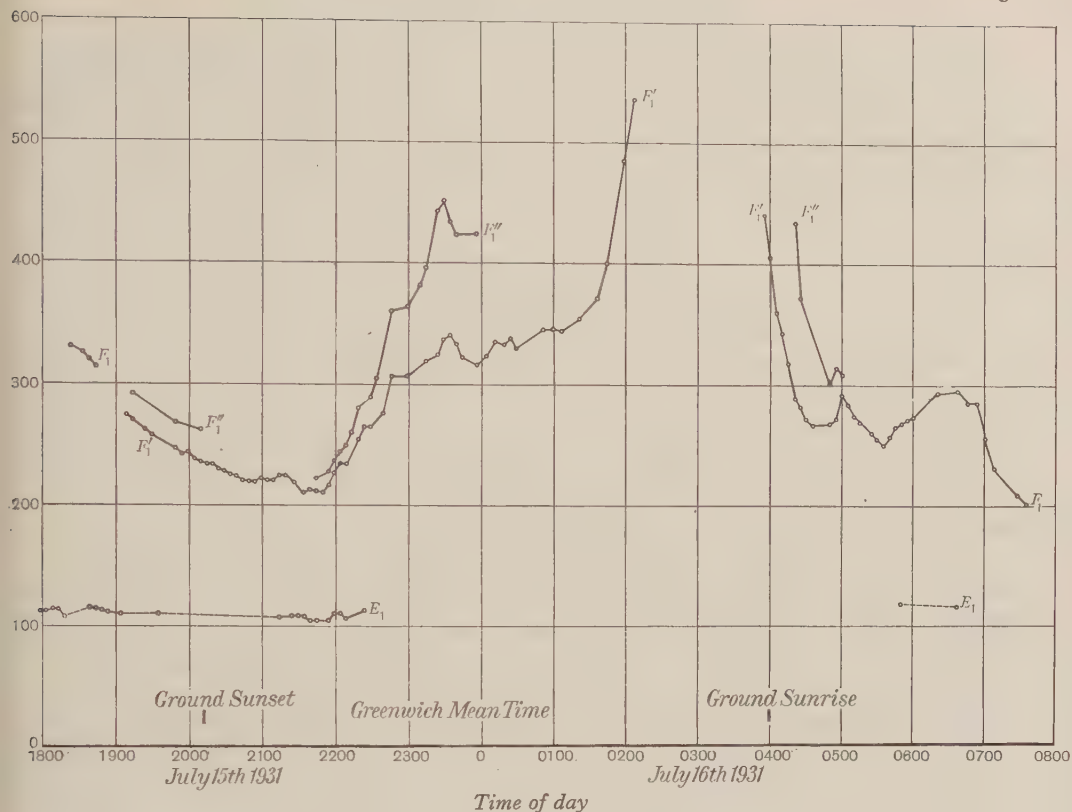
Fig. 3.

§7. DISCUSSION OF RESULTS

Our experience with the use of the group-retardation method has enabled us to compare the relative advantages and disadvantages of this method and the frequency-change method previously used. These points are discussed immediately below under various heads (A) to (E).

(A) So far as accuracy of the determination of the echo-time $\int \frac{ds}{U}$ is concerned, there is perhaps little to choose between the two methods. In the case of the frequency-change method we count the number Δn of interference maxima and minima produced by a continuous change of emitter frequency Δf, and the retardation time is given simply by $\Delta n/\Delta f$. Now Δn can usually be estimated to 1 per cent. and Δf to a much greater degree of accuracy, so that the overall accuracy is to about 1 per cent.

In the group-retardation method the retardation time has to be measured directly by means of a high-speed oscillograph. It is our experience that the photographic records obtained, when examined under a microscope, permit the determination of this time to about 1 per cent.

6-2

(B) One of the greatest difficulties in making equivalent-height measurements at short distances from the emitting station is that the intensity of the atmospheric waves is relatively small compared with that of the ground waves. This is notably the case during the day-time and when relatively long waves are used. For such conditions the frequency-change method possesses an inherent advantage over the group-retardation method when, as is usually the case, a square-law detector is employed in the receiver. Let us, as an example, consider a typical case in which the intensity of the down-coming wave is $\frac{1}{20}$ of that of the ground waves. In the case of the frequency-change method the interference maxima will be to the interference minima in the ratio $(1 \cdot 05 \; 0 \cdot 95)^2$, so that the intensity of the combined signal varies by about 20 per cent., an amount which readily permits an accurate measurement of Δn.

For the same conditions, when the group-retardation method is used and the echo and ground signals have to be detected separately, the ratio of their amplitudes is $(\frac{1}{20})^2$ or $0 \cdot 002$, a value which would make the recognition of the echo exceedingly difficult if not wellnigh impossible.

We believe that this inherent insensitivity of the group-retardation method for weak echo conditions has been the cause of the restriction of its use to short waves, whereas, with the frequency-change method, observations on a wide range of wave-lengths have been carried out.

(C) The group-retardation method possesses very great advantages over the frequency-change method for conditions, often encountered at night, in which multiple echo signals are obtained. As has been seen from the records above, such echoes are readily identified if the pulses sent out are sufficiently short. The same convenient separation of multiple signals can be delineated on a cathode-ray oscillograph and their variations in intensity can be watched continuously, accurate records being taken by means of the high-speed oscillograph when desired. It is true that, in the case of the frequency-change method, a long experience in examining the records may enable us to unravel a case in which, say, two sets of down-coming waves are present, but it is certain that more complicated cases where there are many echoes would defy even the most experienced. Moreover, during the course of an experiment with the frequency-change method, the nature of the changes that are occurring are not known until the Einthoven galvanometer records are developed, whereas, with the dual representation scheme we have adopted for the group-retardation method, we are aware of the nature of the intensity and retardation of the echoes all the time, and no interesting features are necessarily missed.

(D) Another advantage of the group-retardation method with the quasi-stationary delineation of the received echoes described above is to be found in connection with the measurement of the maximum ionization-content of the Kennelly-Heaviside layer (region E). As has been mentioned there are two ionized regions in the upper atmosphere at which waves are reflected, the upper region F being richer in ionization than the lower region E. If, therefore, a series of mean frequencies are used for equivalent-height determinations, a discontinuity is found

in the curve representing equivalent height as a function of frequency. The discontinuity corresponds to the frequency which just penetrates the lower region. When the critical frequency is known it is possible, if certain assumptions are made, to calculate the maximum ionization-content in this lower region.

To find the critical frequency it is necessary to obtain data for the equivalent-height curve for a selected range of frequencies in which the discontinuity is expected. Such observations have been made with the frequency-change method, but this method is clearly somewhat laborious.

Since, however, it is found that the variation of ionization with time is usually gradual, the change-over from reflection at one region to reflection at the other can very conveniently be studied by the use of the group-retardation method with quasi-stationary delineation. For example, if the ionization in the lower region is increasing, the pulses can be sent out on a radio-frequency slightly higher than that corresponding to the critical value. The change-over from reflection at the upper region to reflection at the lower can subsequently be noted when it occurs.

(E) It is clear that, in all problems in which it is necessary to know the relative phases of the ground waves and the down-coming waves, the frequency-change method possesses enormous advantages when compared with the group-retardation method. The photographic records obtained in the prosecution of the former method show clearly the variation of the phase of the down-coming waves both when the emitted frequency is constant and when its value is continuously altered. From such records it has been possible to study the temporal variation of the optical path of the atmospheric waves. By means of similar records made with differently oriented frame aerials it has also been feasible to determine completely the specification of the polarization of the down-coming waves. Such determinations have not so far been found possible with modifications of the group-retardation method.

§8. ACKNOWLEDGMENTS

APPENDIX

A LINEAR TIME-BASE OSCILLATOR FOR CATHODE-RAY OSCILLOGRAPHY

The use of the neon tube as a relaxation oscillator for producing a linear and unidirectional time-scale on a cathode-ray oscillograph has been previously described by various writers*. The fundamental circuit from which such oscillators are derived is shown in figure 4.

* G. I. Finch, R. W. Sutton and A. E. Tooke, *Proc. Phys. Soc.* **43**, 502 (1931); and E. L. C. White, *Proc. Camb. Phil. Soc.* **27**, 445 (1931).

Such a circuit has the disadvantage that the time-base departs considerably from linearity unless a current-limiting device, such as a saturated diode, is used in place of the resistance R. Also, the difference between the critical voltages of a neon tube is too small to provide sufficient voltage-swing for the time-base, particularly when medium-voltage oscillographs are being used.

It should be noted that if sufficiently high potential is used for the battery B, figure 4, the variation of the potential across C with time is sensibly linear between the values of the striking and extinguishing potentials. Thus we have found that, if, with an ordinary 200-volt tube, the potential be increased to anything above 400 volts, the potential-change with time is sufficiently linear for practical purposes.
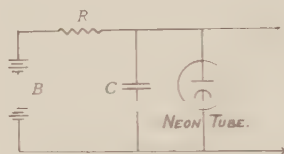
Fig. 4.
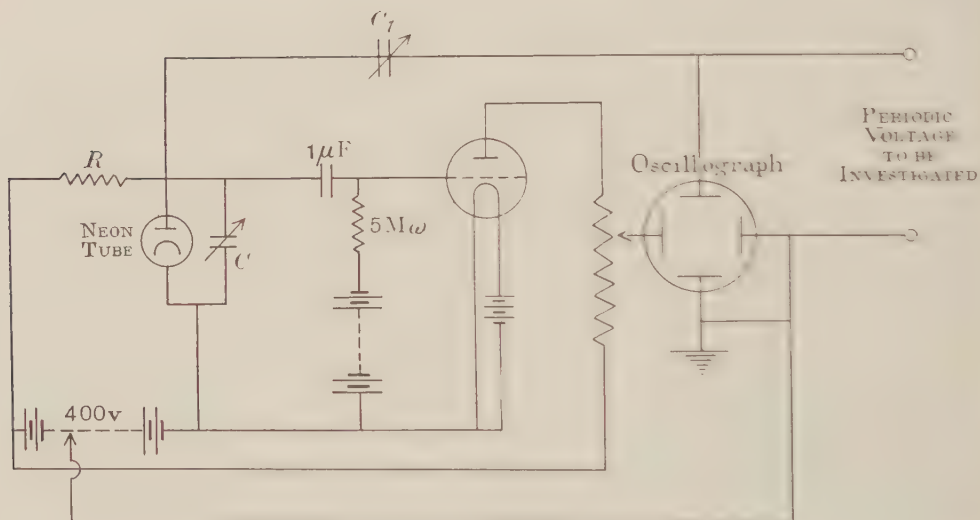
Fig. 5.

In order to overcome the difficulty that, with the simple circuit of figure 4, we are limited to a voltage-swing equal to the difference between the striking and extinction voltages, a valve amplifier may be added.

A suitable circuit incorporating both these modifications is shown in figure 5. We have found this to be very satisfactory and extremely easy to adjust, if attention is paid to the following points:

(*a*) The time-constant of the grid circuit of the amplifier must be of suitable value for the frequencies to be used. The values given are satisfactory down to less than 50 ~.

(*b*) The relation between the grid potential of the triode and the output potential across the anode resistance must be linear over the range of potentials used. An LS 5 valve gives such a relation for grid potentials from −60 to −10 volts, with a 1-megohm resistance, the actual voltage amplification being 5.

The voltage variation applied to the cathode-ray oscillograph may be readily varied to the desired value by a suitable tap on the anode resistance. In cases where it is desired to synchronize the time-base oscillator with any periodic voltage under investigation, the locking condenser C_1 may be used.

The circuit described is not difficult to maintain with satisfactory insulation for, since the linearity of the voltage-variation is not effected by the use of a non-linear resistance, spurious leaks are relatively unimportant. A further advantage is that the time-base frequency is little affected by adjustment of the operating conditions of the oscillograph, as for example, when a change is made from an anode potential of 1500 volts, suitable for visual observation, to one of 3000 volts, for photography.

THE EFFECTIVE MASS OF FLEXIBLE DISCS AND CONICAL DIAPHRAGMS USED FOR SOUND-REPRODUCTION

By N. W. McLACHLAN, D.Sc., M.I.E.E., F.Inst.P.

ABSTRACT. An experimental method of measuring mechanical impedance, which is used to ascertain the effective mass of vibrating discs and conical diaphragms, is described. It is shown that the effective mass of a circular aluminium disc vibrating in air is zero at the centre-stationary and centre-moving modes. At a centre-stationary mode the effective mass attains a positive maximum before the zero value and a negative maximum thereafter. From the shape of the curves for a disc it is possible to interpret those obtained for conical diaphragms. In the latter case the curves depend upon the apical angle of the cone. Three types are illustrated: (*a*) a large cone having ψ equal to 160°; (*b*) a loud-speaker cone having ψ equal to 90°, with reinforced edge; (*c*) a loud-speaker cone having ψ equal to 90°, mounted on a rubber annulus. In case (*a*) the disc characteristics are clear, whilst in (*b*) and (*c*) the behaviour is modified owing to the greater degree of conicality. The rubber surround acts as an auxiliary resonant diaphragm, introducing an abrupt change in the effective mass. Finally, the effective mass of a rigid disc vibrating in a finite and in an infinite baffle is considered.

§1. INTRODUCTION

THE use of conical diaphragms in the design of modern loud-speakers has stimulated scientific interest in their physical behaviour. When a diaphragm is driven by an alternating force, its effective mass varies throughout the acoustic register. During vibration in vacuo the effective mass depends upon the relation between the elastic and inertia forces and also upon internal losses. In air an additional loss is imposed owing to the radiation of sound, whilst an increase in effective mass occurs owing to divergence of sound waves from the diaphragm as source. The latter is known as the "accession to inertia" and has been treated in former publications[*]. The problem is best approached by consideration of the ideal case of a flexible annularly-driven homogeneous circular disc vibrating in vacuo, there being no loss whatsoever. This has been treated by A. G. Warren[†]. The curves of figure 1 show the apparent or effective mass M_e from zero frequency upwards. At $f = 0$, M_e is the natural mass of the disc. Owing to the interaction of elastic and inertia forces, M_e increases with the frequency and approaches infinity asymptotically when the disc vibrates with its centre stationary.

Thereafter it increases from $-\infty$ until at the first centre-moving symmetrical mode (one nodal circle) $M_e = 0$, i.e. the sum of the elastic and inertia forces is zero.

M_e
f

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* Lord Rayleigh, *Sound*, **2**, 162 (2nd edition, 1894); N. W. McLachlan, *Phil. Mag.* **7**, 1011 (1929); **11**, 1137 (1931).

† *Phil. Mag.* **9**, 881 (1930).

Passing through zero M_e becomes positive until the next centre-stationary mode is reached, when the former cycle is repeated.

An approximate analogy to the first centre-stationary mode can be drawn from the impedance of the parallel condenser inductance circuit of figure $2a$. The effective inductance is given by $L_e = L/(1 - \omega^2 LC)$. It increases from L at $f = 0$ $\quad L_e, L/(1-\omega^2 L$ to $\pm \infty$ at $\omega^2 LC = 1$ or $1/f = 2\pi \sqrt{(LC)}$, where the centre-stationary condition is simulated.
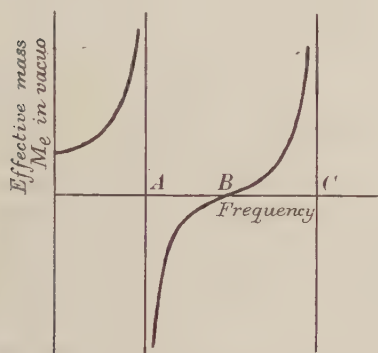
Fig. 1. Curve showing variation with fre-
quency in effective mass of annularly
driven loss free circular disc vibrating
in vacuo.
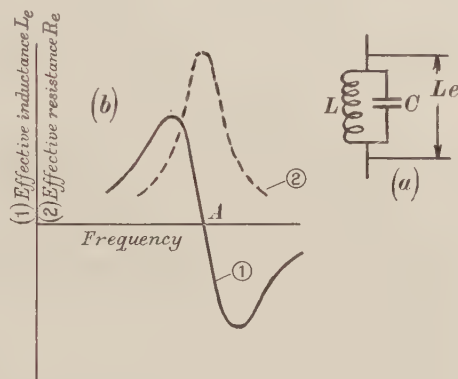
Fig. 2. (*a*) Parallel resistanceless *LC* circuit.
   (*b*) (1) Effective inductance of circuit in $2a$,
   (2) effective resistance of circuit in $2a$,
   when loss occurs.

The effect of a resistance in series with the coil is to alter the effective inductance curve from the form of figure 1 to that shown in curve 1, figure $2b$. The effective inductance cannot become infinite owing to the energy required to balance the loss. Similarly in the case of an actual disc vibrating in air, the effective mass cannot become infinite. Moreover, in practice there is central motion at a so-called centre-stationary mode arising from the supply of energy required to sustain vibration and overcome the internal and acoustic losses*. In the electrical case if R is the a.c. $\quad R$ resistance in circuit, the effective inductance is zero when $\omega^2 = 1/LC - R^2/L^2$, but the effective resistance and the impedance are maxima. The current is, therefore, a minimum, and this corresponds to minimum amplitude at the centre of the disc. The effective inductance has a maximum and a minimum value approximately equally spaced on either side of the zero point. In the disc case this is illustrated clearly in figure 4. It is evident not only that the resistance causes the effective mass to alter from $\pm \infty$ to zero, but that the resonance frequency is altered slightly. In addition to the various losses there is also the accession to inertia, so that the frequencies of the centre-stationary modes would be expected to occur at values slightly different from those found by calculation from the *in vacuo* theory.

* Alternatively, the energy transmitted outwards exceeds that reflected from the edge so that annulment does not occur at the centre; i.e. the centre is not a node but a point of minimum amplitude. This is discussed later in the paper.

By examination of a curve showing the effective mass of a vibrating diaphragm at various frequencies, it is often possible to ascertain the mode of vibration, i.e. whether stationary-centre or moving-centre. We shall consider symmetrical modes of vibration only, since the variation in effective mass of a homogeneous symmetrical diaphragm due to radial modes is substantially zero. It has been found that with actual conical diaphragms having a seam which destroys the symmetry, there is a definite variation in effective mass, but it is small compared with that due to a symmetrical mode. The variation in the magnitude of M_e is a measure of the degree of asymmetry due to the seam and to heterogeneity of the paper.

§2. THEORY OF METHOD OF MEASUREMENT OF MECHANICAL IMPEDANCE

The method of measurement is based on the fact that the mechanical impedance of a vibrating body can be represented by an effective mass in series with an effective mechanical resistance. When the body is attached to some form of electromagnetic drive, the alteration in the effective inductance and resistance is a measure of the mechanical impedance.

Taking the case of an electromagnetic drive having a simple cantilever reed or equivalent elastic member, assumed to act in the same manner as a coil spring: when this is loaded by a diaphragm we obtain*:

$$R_m = C^2 \omega^2 B / \{\omega^2 B^2 + (k_1 - \omega^2 M_e)^2\} \qquad \text{......(1)},$$

$$L_m = C^2 (k_1 - \omega^2 M_e) / \{\omega^2 B^2 + (k_1 - \omega^2 M_e)^2\} \qquad \text{......(2)},$$

R_m where R_m is the difference between the resistances with reed free and reed fixed;

L_m L_m the difference between the inductances with reed free and reed fixed;

B B {the mechanical resistance due to diaphragm loss and sound-radiation; the in-phase force per unit axial velocity *of the driving mechanism*†;

C C {the electromechanical conversion factor; the e.m.f. induced in winding of driver per unit velocity of reed; the force on the reed per unit current (absolute);

k_1 k_1 the coefficient of restitution of reed, i.e. force per unit deflection;

M_e M_e {the total effective mass on reed. This $= m_0 + M_a + M_1$; = (effective mass of reed) + (effective mass of diaphragm) + (accession to inertia).

The relationship between the various mechanical quantities is shown vectorially in figure 3*a*. *B* is the effective load or force component in phase with the axial velocity, whilst $(k_1/\omega - \omega M_e)$ is the effective mass-reactance component in quadrature with the load. When $k_1 > \omega^2 M_e$ the effective mass of the system, including

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* *Phil. Mag.* **7**, 1017–1020 (1929); **11**, 3–4 (1931).

† The axial velocity of a point on the diaphragm varies according to the mode of vibration.

the reed, is negative owing to the reed compliance, whilst it is positive when $\omega^2 M_e < k_1$, i.e. above resonance. The total mechanical impedance is

$$\{B^2 + (k_1/\omega - \omega M_e)^2\}^{\frac{1}{2}}.$$

The analogy with an electrical circuit is obvious.
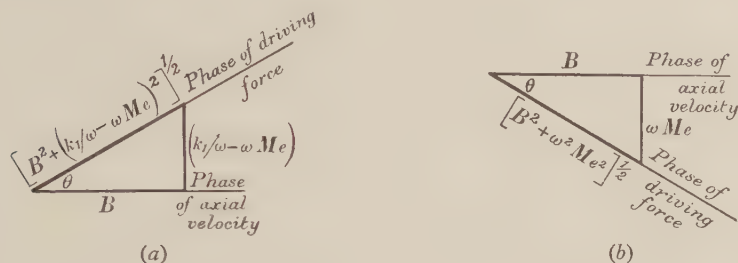
Fig. 3. (a) Vector diagram of reed-driven vibrating system. (b) Vector diagram of coil-driven vibrating system (no constraint of driver).

Solving (1) and (2) we obtain

$$B = C^2/R_m\,(1 + z^2) \qquad\qquad ......(3),$$

and the total effective mass of the system is M_e, where

$$M_e = \{k_1 - (C^2/L_m).z^2/(1 + z^2)\}/\omega^2 \qquad\qquad ......(4),$$

and

$$z = \omega L_m/R_m = \tan\theta.$$

In the case of a diaphragm driven by a moving coil $k_1 = 0$, and the total effective mass of the structure, including the coil, the accession to inertia and the influence of the surround is given by

$$M_e = -\,(C^2/\omega^2 L_m)\,z^2/(1 + z^2) \qquad\qquad ......(5).$$

The vector diagram for this case is shown in figure 3 b. The quantity

$$z^2/(1 + z^2), \text{ or } 1 - \cos^2\theta$$

includes flexibility of the diaphragm and various losses. When these are negligible z^2 is large compared with unity so that $z^2/(1 + z^2) \doteqdot 1$. Then the formula reduces to $-\,C^2/\omega^2 L_m = M_e$, which is identical with that used in a former paper devoted to the measurement of accession to inertia*. When the loss component B is comparable with or greater than the mass reactance ωM_e, the quantity $z^2/(1 + z^2)$ is appreciably less than unity—see table 1 near the resonance frequency 120·6 $\sim$.

§3. DATA FOR COIL-DRIVEN CIRCULAR ALUMINIUM DISC

A coil 2·5 cm. in mean radius, wound on a paper former having a free length of 2·5 cm., was securely fixed coaxially to an aluminium disc 10 cm. in radius and 0·055 cm. thick. The coil was situated in the radial field of a circular electromagnet

* *Phil. Mag.* 11, 1139 (1931), expression (2).

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and the disc was suspended freely by several thin elastic threads, the natural frequency in the absence of the field being about 3 ∼. Bridge measurements* of the inductance and resistance of the coil, free and fixed, were taken over a certain frequency band. Some of the observed and deduced data are given in table 1, whilst a curve showing the effective mass is plotted in figure 4.

Table 1: Showing data for computing the effective mass of an aluminium disc.

Radius of disc, 10 cm. Thickness, $5·5 \times 10^{-2}$ cm. Mass, 47 gm. Mass of coil, former, adhesive and connecting wires, 7·8 gm. Mean radius of coil, 2·5 cm. D.C. resistance of coil without leads, 0·95 ohm. Accession to inertia at zero frequency, 3·5 gm. $C^2 = 2 \times 10^4$. No baffle.

Frequency ($\sim$)	L_m, motional inductance (henry)	R_m, motional resistance (ohms)	$z = \omega L_m / R_m = \tan\theta$	$z^2/(1+z^2)$	M_e, effective mass (gm.)	Remarks
0	—	—	—	—	+ 58·3	Natural mass (47 + 7·8 plus accession to inertia (3·5)
60	$- 6·5 \times 10^{-4}$	10^{-2}	− 24·5	1·0	+ 216	First centre-stationary mode at 69 ∼
75	$+ 7·1 \times 10^{-4}$	3×10^{-2}	+ 11·1	1·0	− 127	
100	$+ 4·2 \times 10^{-4}$	7×10^{-2}	+ 3·8	$9·4 \times 10^{-1}$	− 113	
120	$+ 10^{-2}$	23·3	$3·2 \times 10^{-1}$	9×10^{-2}	$- 3·2 \times 10^{-?}$	First centre-moving mode. One circle of minimum amplitude at 120·6 ∼
120·6	0	39·5	0	0	0	
125	$- 7·2 \times 10^{-3}$	$6·3 \times 10^{-1}$	− 9·0	1·0	+ 4·5	
150	$- 1·45 \times 10^{-3}$	4×10^{-2}	− 34·6	1·0	+ 15·4	
200	$- 4·5 \times 10^{-4}$	6×10^{-2}	− 9·5	1·0	+ 27·5	

Starting from $f = 0$, where limiting motion is assumed, M is the sum of the natural mass (disc plus coil) and M_1 the accession to inertia. Since no baffle was used throughout the experiments, M_1 in the neighbourhood of zero frequency is half its value with an infinite baffle †. The first centre-stationary mode occurs in the neighbourhood of 69 ∼ (A), and the first centre-moving mode at 120·6 ∼ (B). Near the centre-stationary mode M_e attains a positive maximum and then falls to a negative minimum as foreshadowed in the argument associated with figure 2b. The amplitude of the motion is a minimum when $M_e = 0$, since the effective mechanical resistance and impedance are then maxima.

The arithmetical values at the maxima are much in excess of the natural mass of the disc and coil. Rising from the negative maximum the effective mass becomes zero in the neighbourhood of the first centre-moving symmetrical mode at 120·6 ∼. Thereafter it increases, but near 260 ∼ there is a minimum value due to some irregularity. The second centre-stationary mode occurs at 410 ∼ (C). The second centre-moving mode occurs at 484 ∼ (D). The third centre-moving mode, which should occur in the neighbourhood of 1100 ∼, is absent. Considered as a separate

* Phil. Mag. 11, loc. cit.
† McLachlan, J. Inst. E.E. 69, 613 (1931). It is assumed, of course, that the disc is actually moving.

vibrator, the disc within the coil has a mode near that of the outer annulus. More-over, mutual interference suppressed both modes. When allowance is made for the mass of the coil and the portion of the disc within it, the frequencies of the modes determined experimentally are in close agreement with those calculated from Warren's analysis.

From the shape of the curve of figure 4 and the known behaviour of a flat disc it is possible to interpret the more complicated behaviour of conical diaphragms used for loud speakers, by examination of their effective mass curves.
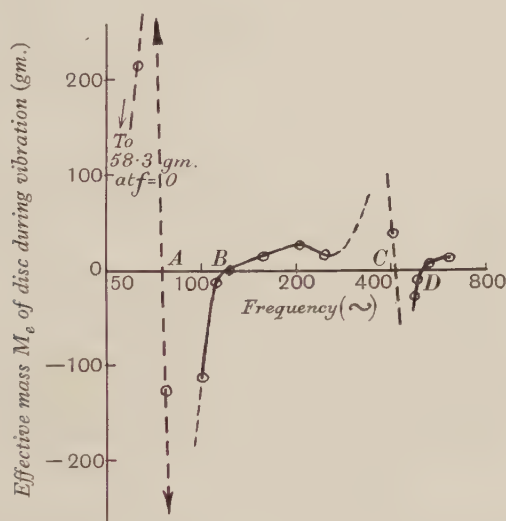
Fig. 4. Curve showing variation with frequency in effective mass of coil-driven aluminium disc 10 cm. radius. *A*, first centre-stationary mode; *B*, first centre-moving mode; *C*, second centre-stationary mode; *D*, second centre-moving mode.

In table 1 C^2, the electromechanical conversion factor, has been assumed con-stant for simplicity. Actually, there is a variation with frequency due to the iron loss in the circular magnet. The value of C^2 can be found at various frequencies by suspending the coil alone by fine elastic threads and measuring its inductance free and fixed. Then from (5), if M_c is the mass of the coil, $C^2 = -\omega^2 L_m M_c$.

M_c

§4. DATA FOR LARGE CONICAL DIAPHRAGM

A flat disc is a cone with a plane apical angle of 180°. The radius being kept constant, if the angle is decreased to, say, 160° the disc characteristics will not be entirely lost. As the angle is reduced the properties peculiar to a conical diaphragm will assert themselves. At the extreme end of the scale when the angle is zero we have a cylinder, and its characteristics will be evident in a conical diaphragm, particularly when the apical angle is small. The increase in stiffness due to conic-ality is remarkable. In the cone treated in this section, the stiffness at the first centre-moving mode ensuing from an angle of 160° is equivalent to that of a disc

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of the same material nearly 16 times as thick*. Data pertaining to this cone are given in table 2, and the effective-mass curve is plotted in figure 5. The diaphragm was coil-driven, the conditions being similar to those for the aluminium disc. Referring to figure 5 and starting at 100 ~, we see that the effective mass behaves with increase in frequency as it does in the case of the aluminium disc.

Table 2: Data for coil-driven paper cone.

Natural mass of coil and diaphragm	$(7.84 + 40.16)$, $= 48$ gm.
Radius at base of cone	16.7 cm.
Plane apical angle of cone	160°
Condition at edge	Free, supported by three thin elastic threads
Natural frequency of cone on threads without magnetic field	Less than 3 ~
Baffle	None
Class of paper	Stiff Whatman 5×10^{-2} cm. thick
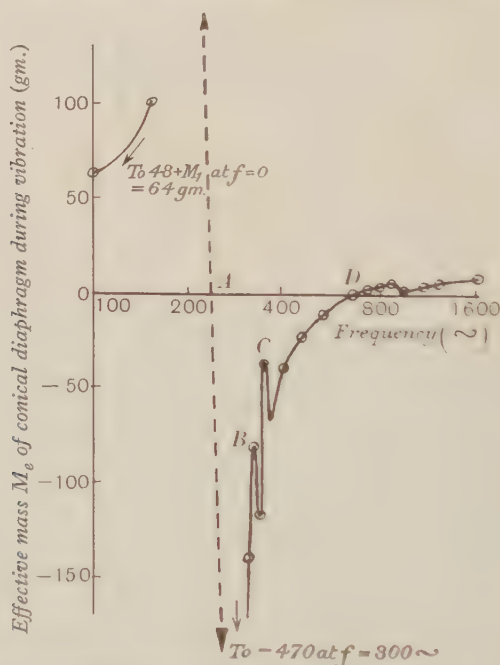
Fig. 5. Curve showing variation with frequency in effective mass of coil-driven cone ($\psi = 160°$) 16.7 cm. in radius. A, first centre-stationary mode; B, first centre-moving mode; C, second centre-stationary mode; D, second centre-moving mode.

The first centre-stationary mode occurs about 240 ~ (A). Following the curve as it rises from a negative value of 470 gm. at 300 cycles, we are, from the similarity with figure 4, tempted to think that the first centre-moving mode (one nodal circle) occurs at 664 ~ (D). Strangely enough, however, tests with lycopodium

* This is treated in the *Philosophical Magazine*, **12**, 771 (1931).

powder revealed two nodal circles at 664 ∼ but only one at 350 ∼ (C). In both cases there were radial nodes also. R_m, the motional resistance, reached a decided maximum at C, so that we can regard this as the first centre-moving mode of vibration. A button microphone fitted with a thick wire feeler was drawn radially over the surface from the centre outwards. At 350 ∼ there was a position of minimum motion, but no actual position of complete rest.

I have shown elsewhere* that when transmission and radiation losses occur there cannot be a position of complete rest on a vibrating diaphragm. The so-called nodal circle is actually a line of minimum amplitude. At any concentric circle on the diaphragm the energy transmitted outwards exceeds and is out of phase with that reflected inwards from the edge. The nearer the centre the greater the ratio (transmitted energy)/(reflected energy). Consequently, the smaller the radius of the so-called nodal circle the greater is the minimum amplitude of vibration. This effect was obtained with a centrally-driven aluminium disc also. Sand lay placidly on a circle when the amplitude was small, but danced vigorously when it was large.

The expression "nodal circle" has been used in connection with conical diaphragms, but the actual lines of minimum amplitude traced by the lycopodium powder were quite irregular and sometimes discontinuous.

Approximate measurement for the two modes gave ratios of the radii of the circles of minimum amplitude to that at the edge of the diaphragm as shown in table 3. The case of a flat circular disc is added for comparison. It will be seen that there is a close resemblance between the unloaded disc and the loaded cone, which shows that in this respect the two types of diaphragm have something in common.

Table 3: Comparison of radial nodes of cone and disc.

Mode of vibration	Cone (coil-loaded)	Disc (unloaded)†
First centre-moving	0·66	0·68
Second centre-moving	0·45, 0·8	0·39, 0·84

† In the absence of radial nodes.

It is clear that the shape of the diaphragm during vibration is determined partly by (a) transmission loss, (b) acoustic load, and (c) reactive load due to accession to inertia. If the preceding impedance measurements are repeated in vacuo, (b) and (c) disappear. Moreover, the shape of the diaphragm during vibration in vacuo is different from that in air, more especially at a centre-moving mode where the acoustic load is large. Thus the difference between the effective mass in air and that in vacuo is not the true value of the accession to inertia, although it may be an adequate approximation for certain purposes‡.

* Letter to *The Wireless Engineer and Experimental Wireless*, October, 1931.
‡ *Phil. Mag.* **12**, 814 (1931).

§5. DATA FOR LOUD-SPEAKER DIAPHRAGMS

The result of measurements on two conical loud-speaker diaphragms (1) with the edge reinforced by a narrow annulus of presspahn to prevent radial modes, (2) with a rubber surround, are portrayed graphically in figure 6. In case (1) starting from zero frequency, where M, is the natural mass plus the accession to inertia, the effective mass gradually rises owing to increase in the accession to inertia up to 200 ~. Thereafter the rise is chiefly concerned with the elastic and inertia forces of the diaphragm structure, the action being similar to that of the aluminium disc and the flat conical diaphragm of the preceding section. The maximum at B_1 portends approach to a centre-stationary mode, whilst the minimum and subsequent rise at C_1 indicates the first centre-moving mode.
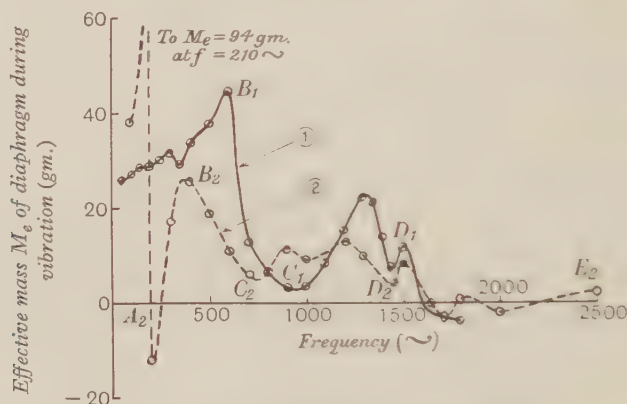
Fig. 6. Curves showing variation with frequency, in effective mass of coil-driven conical paper diaphragms. Radius $=12\cdot2$ cm.; $\psi=90°$. Curve 1. Edge of diaphragm reinforced by narrow press-pahn annulus to suppress radial modes. Curve 2. Edge of diaphragm bent over and supported by rubber annulus.

Beyond C_1 the effective mass rises until at about D_1 the second centre-stationary mode occurs. Thereafter M_e steadily falls and becomes negative. The second centre-moving mode occurs about 2000 ~. As with the former cone M (not shown graphically) is substantially zero. I have indicated elsewhere* that the stresses in cones and discs are entirely different and, moreover, the behaviour during vibration differs too.

Curve 2 relates to a diaphragm similar to that of curve 1 but mounted on a rubber surround. Resonance of the latter per se† introduces a condition at A_2 of the same nature as a centre-stationary mode (the diaphragm being the driving agent in this case) followed by a centre-moving mode. The remainder of the curve, modified in a degree by the surround, is similar to that of curve 1. The first centre-moving mode is at C_2 and the second about E_2. There are irregularities in each curve, particularly the second, due to asymmetry; for instance, to the seam and heterogeneity of the paper. It is clear, however, that except in the first centre-moving

* See *Phil. Mag.* **12**, 801 (1931).   † *Phil. Mag.* **11**, 28 (1931).

mode the diaphragms behave in a somewhat similar manner to the aluminium disc cited in § 3. Moreover, an effective-mass curve is a useful guide to the mechanical behaviour of a vibrating system.

Table 4: Data for conical coil-driven loud-speaker diaphragms.

Radius at periphery of paper cone with reinforced edge (curve 1)	12·2 cm.
Radius at periphery of paper cone with rubber surround (curve 2)	12·2 cm.
Radial width of rubber surround	1·8 cm.
Apical angle of both cones	90°
Natural frequency of reinforced edge cone on elastic threads without magnetic field	Less than 3 ∼
Natural frequency of cone as a whole on rubber surround without magnetic field	About 30 ∼
Baffle	6 ft.²
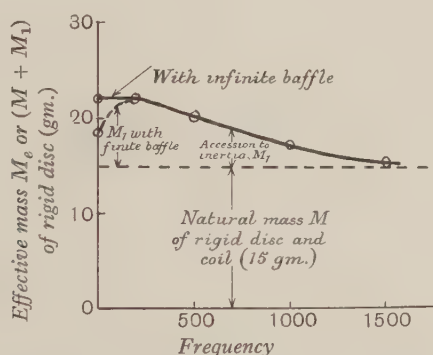
Fig. 7. Curves showing variation with frequency in effective mass of freely supported centrally driven rigid disc, (*a*) with infinite baffle, (*b*) with finite baffle 6 ft. square.

§6. EFFECTIVE MASS OF VIBRATING RIGID DISC

Finally it may be useful for purposes of comparison to give the case of a coil-driven rigid disc vibrating in (i) an infinite baffle, and (ii) a finite baffle. The former was treated elsewhere some years ago* and the data obtained from column 2 of table 2 of the former paper are shown graphically by the full-line curve of figure 7. The influence of a finite baffle, say, 6 ft. square, is shown by the dotted curve rising from $M_e = 18\cdot5$ gm. at $f = 0$. Here, as stated previously†, M_1 is reduced to one-half its value with an infinite baffle. With a baffle 6 ft. square and a disc 10 cm. in radius, M_1 at $f = 0$ is substantially half its value at 200 ∼. So far as rigid discs or diaphragms are concerned, a baffle has a much greater effect on the low-frequency sound-radiation than it has on the accession to inertia.

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* *Phil. Mag.* 7, 1011–1038 (1929).   † See footnote (†) on p. 92.

DISCUSSION

Dr L. E. C. HUGHES: We owe much to Dr McLachlan's exposition of the mode of operation of large-diaphragm loud-speakers, but although his publications have been spread over a period of years there seems to be little desire to use methods other than empirical in the design of commercial models. The method of measuring mechanical impedances described by Dr McLachlan is indirect and involves considerable calculation, which is inconvenient when a large number of measurements have to be undertaken in economic research. A direct method has been described by E. Mallett and R. C. G. Williams*. The present meeting seems an appropriate moment for describing an empirical modification of an electro-dynamic method due to K. Kobayasi† which I put forward for measuring the driving impedance of gramophone reproducers, but which can be modified easily for other applications. The impedance-machine consists of a pot-magnet, in the gap of which moves a former carrying two coils. One of the coils takes the driving-current while the other developes an e.m.f. proportional to the velocity of motion. The ratio of the current to the e.m.f. is a measure of the mechanical impedance $\bar{Z}$ of the system. It was found that, when comparatively heavy masses were attached to the former, the locus of $\bar{Z}$ was a straight line and increments of $\bar{Z}$ were linear with increments of mass. A heavy former of impedance $\bar{Z}$ is therefore used. To this is attached a known mass which alters the impedance to $\bar{Z}_1$. If, when the unknown impedance is attached in place of this, the driving-impedance becomes $\bar{Z}_2$, then the unknown impedance is obtained vectorially from $(\bar{Z} - \bar{Z}_2)(\bar{Z} - \bar{Z}_1)$ in terms of the calibrating mass, which, of course, is analogous to an inductive reactance. For measuring the currents and e.m.fs. a Pederson potentiometer was developed and was manufactured by Messrs Tinsley. This was capable of giving sufficiently fine adjustments, about 1 in 500 up to 5000 ~, for determining the vector differences with suitable accuracy. By heavily loading the driving circuit so that the driving-current remains constant with small changes of mechanical impedance, the readings of the potentiometer dials may be plotted directly on graph paper with one slide-rule calculation. Such a method has perhaps more appeal to an engineer than to a physicist; unfortunately the method was not used except for demonstrating its validity.

Mr R. S. WHIPPLE enquired whether the author had used Coker's method to locate the nodes in a celluloid diaphragm.

Prof. F. L. HOPWOOD suggested that cellophane would be more suitable than celluloid for the purpose proposed by Mr Whipple as it is doubly refracting and very homogeneous, and can be obtained in various thicknesses. The variation in accession to inertia with change of frequency depends upon the reciprocating flow of air across the nodal lines of the diaphragm, between places in opposite phases of vibration. To infer that a body always gives its fundamental tone when struck is not justifiable. Two steel balls one inch in diameter when tapped together give an audiofrequency sound, although the fundamental frequency of each is 136,000 ~.

* *J. Inst. E. E.* **68**, 560 (1930).   † *Tohoku University Reports* (1929) and *I. E. E. Japan* (1928).

Mr T. Smith remarked that Coker's method is applicable only under static conditions. It could not be extended to vibrating diaphragms without elaborate equipment designed to restrict observation to a particular phase of the strain cycle.

Mr A. G. Warren: Reference has been made to other methods of measuring the effective mass of the diaphragm. The mechanical system is so complicated that I cannot but feel distrustful of any method which involves disturbing it by the connexion of any subsidiary measuring device. The author's method avoids this; measurements are made with the diaphragm under normal working conditions.

Mr G. A. V. Sowter (communicated): I should like to mention that the author devised the present method some years ago when he was unable to test it. In 1927 he lent me a bulky manuscript prepared in 1926 and dealing with the theory of moving-coil apparatus. Two parts of this, dealing with the theory of a coil-driven rigid disc in an infinite baffle, were eventually published*. Another part contained the equations and method of measuring mechanical impedance described in the present paper. Dr McLachlan and myself have taken some 20,000 experimental observations on diaphragms, and several papers dealing with the subject have appeared from time to time in the *Philosophical Magazine* during the present year. The method described by Dr Hughes necessitates structural alteration of the diaphragm which will affect its mechanical properties and therefore its effective mass. Also the measurement of mechanical resistance seems to introduce complications and possibilities of appreciable error. Dr McLachlan's method as applied to moving-coil loud-speakers is unique since the diaphragm is tested under actual working conditions. It is a very simple method since R_m and L_m are determined in routine tests of electrical impedance, and M_e is merely a matter of arithmetic.

Author's reply: In criticizing my method of measurement Dr Hughes fails to recognize that in modern science many highly accurate methods are indirect. Objection to an indirect method is akin to refusing payment by cheques or bank-notes in lieu of gold. As Messrs Warren and Sowter point out, the alternative methods would involve structural changes, thereby vitiating the results. The proposed alternatives are not without their defects. Mallett and Williams clearly state the difficulties of their tuning-fork method and its limitation to 500 $\sim$. The mechanical resistance is found by plotting. Is this direct? To justify directness one spends precious minutes juggling with masses in the pious hope of ultimately finding a winner. It also appears that the auxiliary-coil method necessitates graph paper and a slide-rule. Is this what Dr Hughes means by "economic" research? Despite these little irregularities he says that my method "involves considerable calculation." Now the loud-speaker is used as it stands; L_0, R_0, C^2 and $z^2/(1+z^2)$ are extracted from tables or curves; L_1, R_1 are measured simultaneously and L_m, R_m are found immediately by subtraction. The calculation of effective mass and resistance is simple and the total time for all the operations at a given frequency is less than 10 minutes. This method has the advantage that the mass and resistance

* *Proc. R. S. A.* **122**, 604 (1929); *Phil. Mag.* **7**, 1011 (1929).

components are found separately. In Dr Hughes' method a "lumped impedance" is found from which the two components are derived roughly, by an approximation. This can hardly be regarded as accurate measurement.

My method can be extended to pick-ups and other appliances by aid of a stiff coil of small radius ending in a conical spider with suitable means of attachment. Some form of elastic centering device can be used if desired.

Mr Whipple's suggestion has not been tried owing to lack of apparatus. A stroboscopic effect could be obtained by interrupting the polarized light with a Kerr cell at a frequency slightly different from that of the vibrating diaphragm. This overcomes the difficulty mentioned by Mr T. Smith and would be an interesting experiment if the apparatus were available.

The sound concomitant with Prof. Hopwood's bicycle ball experiment is due to the initial "impulse." This covers the entire frequency spectrum, and the ear picks out the audible portion. It would be of interest and use to know how long a pure audible tone must be sounded before it ceases to be merely an impulse or noise. I imagine the time required would be of the order of 10^{-1} sec.

DEMONSTRATIONS

"The Effect of Mechanical Disturbance on a Neon Lamp." *Demonstration given on October* 16, 1931, *by* T. J. DILLON, M.Sc., and C. M. LOVETT, B.Sc., London (Royal Free Hospital) School of Medicine for Women.

The object of this demonstration was to show that, in certain circumstances, a discharge through a neon lamp could be temporarily stopped on the application of a mechanical disturbance or impulse.

A neon lamp was set up in a Pearson-and-Anson flashing circuit. The circuit conditions were so arranged that the smallest current which would support a steady discharge was flowing through the lamp. In the demonstration circuit a lamp of osglim beehive type, with its series resistance removed, was used in parallel with a condenser of capacity $4\,\mu$F, and a short-period galvanometer with lamp and scale could be connected in series with the lamp for reading the current. The lamp was mounted, in an ebonite ring, on a stand firmly clamped to the edge of the bench. The mechanical disturbance was produced by a loaded bar pendulum placed so that at the bottom of its swing it hit the stand on which the lamp was supported.

It was first shown that the magnitude of the impulse necessary to stop the discharge depended on the value of the current flowing through the lamp. The sensitivity was shown to alter with the direction of the impulse in a way that suggests that the effect may be due to the sudden movement of the electrodes relative to the glow. For the purpose of demonstrating that an impulse above a set value had been applied to the lamp, a second similar lamp was placed in parallel with the first, this second lamp lighting up when the glow had ceased in the first.

Many series of observations have been taken, with different types of lamp and condensers of varying capacity, showing how the applied momentum necessary to stop the glow increased with the current, until a value was reached above which the discharge could not be stopped by an impulse alone. In the most sensitive state the lamp could be blown out, and would respond to slight mechanical disturbances due to footsteps and to outside traffic.

(*a*) A contrivance for demonstrating the law of errors; (*b*) A new type of surface-tension-meter; and (*c*) A new type of static electrometer*. *Demonstrations given on November* 20, 1931, *by* Prof. KERR GRANT, M.Sc.

(*a*) *A contrivance for demonstrating the law of errors.* The instrument consists of a number of short metal bars loosely mounted on a spindle with a cast-on wheel by means of which the spindle and bars can be set in rotation. Another light bar mounted on a spring permits of the simultaneous arrest of all the bars, which are painted red on one side, black on the other. The experiment is equivalent

* A fuller description of these instruments will appear in the *Journal of Scientific Instruments.*

to tossing a handful of twenty coins, and its repetition enables a frequency-distribution to be determined.

(*b*) *A new type of surface-tension-meter.* The instrument depends upon the force exerted by surface-tension on a lamina or vane partly within and partly without a narrow channel in the liquid. This force is balanced by gravity or torsion in a light balance carrying the vane. Variations of surface-tension, the production of monomolecular layers and other phenomena can be demonstrated optically to large audiences. The sensitivity is of the order of 1 dyne but can be made much higher.

(*c*) *A new type of electrometer.* The moving element is a very light grid stamped out of sheet aluminium and controlled by a fine strip or wire fastened at top and bottom. There are field-plates similarly constructed. Used idiostatically the instrument shown had a range of 200 volts and followed a square law. Used heterostatically with a field $\pm$ 100 volts on the plates it had a sensitivity of 100 divisions (mm. at 1 metre) per volt with a period of about $\frac{1}{4}$ sec. It is portable and requires no levelling.

REVIEWS OF BOOKS

Newton: the Man, by Lieut.-Col. R. DE VILLAMIL, R.E., with a foreword by Professor ALBERT EINSTEIN. Pp. vi + 111 and frontispiece. (London: Gordon D. Knox, 106 Guilford St., W.C. 1) 3s. 6d. net.

This small but fascinating book is written round Col. Villamil's investigations in the Musgrave Library at Barnsley Park, Gloucestershire, and his most remarkable discovery of the "True and Perfect Inventory of all and Singular the Goods Chattels and Credits of Sir Isaac Newton" at Somerset House. It is divided into three parts: first an extraordinarily interesting and intimate account of Newton's personality and behaviour in everyday life; secondly a copy of the inventory and, lastly, a copy of the catalogue of Newton's library, made *c.* 1760, and discovered by the author at Barnsley Park.

It is to be feared that the portrait is not altogether pleasing. Newton had a cold temperament and, in his later years, was irritable; he was lacking in humour, indifferent to poetry and music, mean in small things and, in his house, had none of the beautiful furniture which characterised his period. On the other hand he was free from conceit, honest and held in the highest esteem by his contemporaries. He died intestate and in consequence an inventory of his belongings was taken by order of the Prerogative Court of Canterbury. His estate was "appraised att the sum of £31,821. 16. 10" but to make an estimate of his savings we must add to this £4000 lost on his holding of South Sea stock, another £4000 given to his great-niece, £500 paid for the translation of the *Opticks* and £200 paid for editorial work on the third edition of the *Principia*. This gives a total approaching £40,000, most of which must have accrued to him from the "profits of the coinage" which formed a large part of his dues as Master of the Mint. In his last years his total income must have exceeded £4000 per annum.

The library comprised 1896 volumes together with "above one hundredweight of pamphets and wast books." The catalogue is remarkable for its large content of theological works and of Greek and Latin classics; English classics are almost entirely absent. Among the scientific books there are no fewer than twenty-two by Boyle but no copy of the *Micrographia*!

Col. Villamil has made a most important addition to *Newtoniana* and the reader is advised to refer to the book for the remarkable history of the library itself and for many interesting sidelights on Newton's life and character.

D. O. W.

James Clerk Maxwell. A Commemoration Volume, 1831–1931. 8vo, pp. 146. (London: Cambridge University Press.) 6s. net.

Some of the ten essays by distinguished physicists which make up this volume were delivered as addresses at Cambridge during the Maxwell centenary celebrations in September. Its publication will help the public to understand something of the high appreciation in which Maxwell's scientific work is held, and the affectionate memories those have of him who were his pupils in the Cavendish Laboratory and are still with us. Sir J. J. Thomson's opening essay of 44 pages is the only one which gives biographical details, and it sums up by expressing "profound admiration for the fineness and strength of his character, for his unselfishness and kindness." The rest of the essay and those of Sir J. Larmor and Sir J. Jeans deal with Maxwell's scientific work and its influence on the progress of physics since his day. Each has something to say on his greatest work— that on the electromagnetic field. Jeans deals in addition with his memoir on the *Dynamical Theory of Gases*, and Larmor touches on his relation to thermodynamics. Planck accounts for the slow progress in Germany of Maxwell's views as to the function of the medium

in electromagnetic phenomena and brings out very clearly the reaction of Maxwell and Boltzmann on each other in the development of the kinetic theory of gases. Einstein dwells on the importance of the transformation of our view of reality as an assemblage of particles into the conception of a continuous field, and Sir O. Lodge traces the development of wireless telegraphy from Maxwell's theory.

Sir H. Lamb tells us about Maxwell's first, and Sir A. Fleming about his last, lecture at Cambridge, and Mr W. Garnett about the apparatus in the Cavendish Laboratory in its early days and those who were set to work with it. Sir R. Glazebrook and Sir A. Fleming describe their own researches under Maxwell, and we gather that Maxwell's method was to plan a student's work carefully for him, and to encourage him to tackle in his own way the difficulties he encountered.

A book by so many authors is bound to have some repetitions in it; and this one is not quite free from discrepancies. One is struck by the paucity of the information about Maxwell's earlier professorships, particularly that at King's College, London, which he held from 1860 to 1865. During these years he published his most important papers, and his reputation for fairness and clear insight into fundamentals was such that there was an incessant demand on his services as referee for papers submitted to scientific societies. The contributions he made to science in this way ought not to be overlooked in any account of his life's work.

C. H. L.

The Mysterious Universe, by Sir JAMES JEANS, F.R.S. Pp. viii + 142. (London: Cambridge University Press.) 2s.

This unpretentious volume, an expansion of the Rede Lecture delivered before the University of Cambridge in November 1930, needs no introduction to members of the Physical Society. It suffices to say that the first edition has completed its first hundred thousand, and that in this second edition the author has brought the scientific matter up to date and has expunged or rewritten certain passages which, in his opinion, were liable to misinterpretation. And, most astonishing fact of all, this crowning instance of infinite riches in a little room may in its new form—pleasing to see and easy to slip into the pocket—be bought for the price of a "Sapper" novel. *Verbum sapienti*.

A. F.

Science and First Principles, by F. S. C. NORTHROP. Pp. xiv + 299. (London: Cambridge University Press.) 12s. 6d.

Mr Northrop's aim is "to determine what contemporary scientific discoveries in many different branches of science reveal, and what all this means for philosophy." That quotation will probably convince most scientists that they do not want to read his book, and that it is only another attempt to show that science means something that no scientist can comprehend. I fear that their conclusion will be right: scientific knowledge will not enable the difficulties of this profound and learned treatise to be mastered, nor will it be increased by mastering them.

The truth is, of course, that philosophers mean by science something quite different from what we mean. To them science is, at any given moment, a definite set of propositions, expressed in words, the import of which is almost or quite independent of the methods by which they were produced. Their meaning is to be found by verbal analysis and comparison with other propositions in which the same words are found. Hence it is that Mr Northrop finds continuity of development, and really very little progress, between the Greeks (who invented so many of our words) and modern times. To scientists science is a form of activity, the "conclusions" of which are merely one part of a never-ending process; they are continually in flux and never fully expressible in words. Since the process rests on the discovery of experimental laws, a conception foreign to the Greeks,

nothing important is common to Greek speculation and modern science. There is no essential reason why those who are interested in scientific science should not also be interested in philosophic science; those who have this double interest will doubtless read Mr Northrop's treatise with great pleasure and profit. But any detailed criticism of it lies wholly without the scope of this journal.

N. R. C

An Introduction to Quantum Theory, by G. TEMPLE, Ph.D., D.Sc. Pp. 196. (London: Williams and Norgate, Ltd.) 12s. 6d.

The student of the new quantum theory who applies himself to the various introductions to this subject may well be surprised that the newcomer changes his appearance somewhat at every new introduction. There is an uncertainty about the character of each new presentation. The treatment in the particular work under review is of an original character and devotes more space than other books of its kind to the methods of matrix and quantum mechanics. The author resembles other writers in that he enters the new territory from a classical base and begins with four very interesting and readable chapters on the principle of duality, the theory of photons, the wave equation and some simple problems. But he soon becomes apologetic for this weakness and proclaims his allegiance to that ascetic school whose walls carry no pictures but which believes that it is impossible to proceed far with any realistic interpretation of physical phenomena. The introduction of this belief into physics is of the nature of the introduction of an important new principle. It merits a careful and not too hurried a treatment. Although the second half of the book is devoted to the methods of the new calculus, the author hurries on to results obtained by it instead of pausing to give some detail about it. This is no adverse criticism of the matter presented, the extent of which is surprising, but one feels that the reading beyond chapter v would have been made much easier if the explanation of the notation had been more extended. The new methods are not so completely new that analogy cannot be traced with those of classical physics, such as that which can be found in extensions of the vector notation. This method helps the new student to make the first plunge and carries him safely to the delights of the deeper waters.

H. T. F.

Der Smekal-Raman-Effekt, by Prof. K. W. F. KOHLRAUSCH. Pp. viii + 392. (Berlin: Julius Springer.) 32 marks, bound 33.80 marks.

The discovery of the Raman effect, early in 1928, opened up a view of research which has almost paralleled the early history of work in X-rays and radioactivity. The rapid development of the subject has been mainly due to the intrinsic interest attaching to the phenomena, and to the extraordinary value of the effect as a tool for fundamental physical and chemical research. Moreover, much of the work can be carried out with relatively simple and standard equipment.

Prof. Kohlrausch lists no fewer than 413 papers on the Raman effect, published between February 1928 and June 1931. The ordering of a proliferation of this magnitude into a consecutive and moderately concise account can have been no light task, even for a writer with the energy and experience of Prof. Kohlrausch, actively engaged as he is in experimental research upon the subject-matter. The task has, however, been very well performed, although the book gives occasional evidences of its necessarily hurried compilation. Prof. Bergen Davis, for instance, generally appears under the alias of "D. Bergen." It would, however, in view of the magnitude of the undertaking, be unjust as well as ungracious to lay stress upon errors of this kind, which are, indeed, almost unavoidable.

The book contains a full account, critical as well as comprehensive, of both the experimental and theoretical sides of the subject, well indexed tables of the known Raman spectra, and a very full bibliography, complete as far as possible to the spring of 1931. There was a real need for a survey of this kind, and Prof. Kohlrausch's book should be assured of a warm welcome.

H. R. R.

Constitution of Atomic Nuclei and Radioactivity, by G. GAMOW. Pp. viii + 114. (London: Oxford University Press.) 10s. 6d.

The investigation of the nucleus is no easy matter, either experimentally or theoretically, and the nuclear electrons are conspicuously refractory to current quantum-mechanical treatment. Nevertheless, notable advances have been made during the last decade in our knowledge of nuclear structure and dynamics. In the book under review Dr Gamow, who has been associated with some of these advances, gives an excellent summary of the present position. The chapter headings are: I, The constituent parts and energy of nuclei; II, Spontaneous disintegration of nuclei; III, Excited states and electromagnetic radiation of nuclei; IV, Artificial transformation of nuclei. The subject-matter is avowedly treated from the theoretical point of view. Experimental details are omitted, but full use is made of the numerical results of the most recent measurements on nuclear phenomena. The book therefore supplements, on the theoretical side, some chapters of the large treatise of Rutherford, Chadwick and Ellis, which was reviewed in these columns last year.

The treatment is throughout exceptionally clear, and non-specialists and macro-physicists will appreciate the marking of the more speculative passages by a distinctive symbol, somewhat resembling the sign commonly used on our English highways to indicate "Dangerous turns ahead."

H. R. R.

L'Atome de Bohr, by LÉON BRILLOUIN. Pp. 363. (Paris: Les Presses Universitaires de France.) 100 fr.

The author of this book holds the view that for a considerable period of time teachers of physics will be obliged to base courses of lectures on Bohr's theory of the structure of the atom and of the emission of spectral lines. With this view we are in substantial agreement, and we must therefore welcome the publication of such a well-balanced survey of the Bohr theory and of the mathematical theorems on which it is based. Incidentally, the book follows the courses of lectures which the author delivered at Wisconsin, U.S.A., in 1928, and more recently at the Sorbonne. It is, indeed, no mere restatement of familiar ideas, and is well worth a close study. The treatment of the magnetic behaviour of the atom is particularly good. The description of the Gerlach and Stern experiment is not good, however, and the diagram of their experimental arrangement needs modification. There is no attempt to describe the methods of wave mechanics, but the bearing of the results of the newer theories upon the old is everywhere adequately expressed.

L. F. B.

Handbuch der Experimentalphysik, Ergänzungswerk-Band 1: *Bandenspektren*, by W. WEIZEL. Pp. xi + 461. (Leipzig: Akademische Verlagsgesellschaft, M.B.H.) 45 marks.

The theoretical interpretation of band spectra of diatomic molecules, begun ten or twelve years ago, has been placed on a new footing and rapidly extended within the last five years. These later theoretical developments have given great impetus to the laboratory investigations of the structures of bands and band systems, the results of which have increased, in number and precision, at a rate which almost bewilders any reader who attempts to keep abreast of them.

The National Research Council Bulletin on *Molecular Spectra in Gases* by Kemble, Birge, Loomis and others, appeared just too soon (at the end of 1926) to include any of the newer theoretical investigations which clarified the subject, but it remains as a valuable record of the knowledge of the spectra of diatomic molecules at a period when no other important book had appeared. In the last year or two, when the theory has reached a more settled state and observations are more completely understood, monographs and larger works have been, and are being, written by several workers on band spectra. Many excellent reviews of the subject and of specific branches of it, such as those relating to heats of dissociation, predissociation, diffuse spectra and continua, active nitrogen, the isotope effect, molecular structure, etc., have appeared from time to time in the *Physikalisches Zeitschrift, Ergebnisse der exakten Naturwissenschaften, Chemical Reviews*, etc. Considerable sections in the newer text-books and handbooks of theoretical and experimental physics, astrophysics and physical chemistry are devoted to molecular spectra; such sections, in fact, appeared in two former volumes, XXI and XXII, of this *Handbuch der Experimentalphysik*. There have also appeared Ruedy's useful monograph *Bandenspektren auf experimenteller Grundlage* (Vieweg, 1930), Kronig's theoretical work *Band Spectra and Molecular Structure* (C.U.P., 1930) and the earlier parts of Mulliken's very thorough report on "The interpretation of band spectra" in *Reviews of Modern Physics*. Several larger works on molecular spectra have been in preparation, and the first of them actually to appear is the present excellent volume by Dr Weizel.

The first half of the book is almost entirely theoretical; it consists of a preliminary section on the molecular model and the Schrödinger equation, and three chapters dealing with: (I) the theory of the terms of diatomic molecules (141 pages), (II) the structure of band spectra of diatomic molecules (67 pages), and (III) the theory of polyatomic molecules (10 pages). The second half of the book consists of a single chapter of 210 pages dealing with the observed bands and band systems of molecules and groups of molecules, namely (1) H_2, (2) He_2, (3) diatomic metallic molecules, (4) diatomic hydrides, (5) diatomic molecules containing one or two of the elements B, C, N and O, (6) diatomic halogen molecules, (7) diatomic halides, (8) monoxides not included in (5), (9) S_2, Se_2, Te_2, (10) other diatomic molecules, and (11) to (14) certain types of polyatomic molecules. A bibliography appears at the end of each of these sections of chapter IV. Included in this chapter are about 50 energy-level diagrams, Fortrat diagrams, potential curves, etc., and 116 tables of numerical data for individual bands, band systems, and molecular constants. The book closes with a very full index occupying 19 double-columned pages; this is especially necessary, because the large amount of numerical data is interspersed throughout the latter half of the book, rather than collected together into an appendix.

The selection, arrangement and presentation of all the matter, both theoretical and observational, are most excellent. Dr Weizel points out that many of the numerical data tabulated have not been critically tested by him, but he is also careful not to let some observations and analyses be included without a note to the effect that they are uncertain. Whilst the data for several molecules are from papers which appeared as late as the middle of 1931, there are a few instances where better were available long before that date, for instance SO (p. 403). In table 133, the v' numeration for the Cl_2 bands is described as Birge's revision of that given in Elliott's 1930 paper. There appears to be some confusion here; the reviewer's recollection of correspondence with Dr Elliott on this point is that Birge's revision applied to the numeration in Elliott's 1929 paper, and that Elliott's own revision in his 1930 paper was somewhat later than Birge's, and was, indeed, accepted by Prof. Birge. In one or two cases a numerical result, actually obtained by the author of an original paper on the analysis of a band system, is attributed to Birge as the compiler of the very useful table of molecular constants in the *International Critical Tables*, vol. V; slips of this kind, however, are quite unimportant.

For the most part the notation employed agrees with that suggested by Mulliken in

1929–30 and now widely adopted. There are important differences, however; for instance in the choice of the numerical subscripts used to distinguish between the spin components of a multiple electronic state and those of a rotational level, and consequently in the designations of the band branches. Weizel's choice is as theoretically consistent as Mulliken's, and the difference between the two is carefully pointed out; nevertheless, to a reader who has become familiar with one scheme the existence of a second, however good, is disturbing, unless and until it in turn becomes the internationally adopted one. In a book of its size, scope and price, well illustrated though it already is, a few reproductions of spectrograms might reasonably have been expected; the lack of these, however, is heavily outweighed by the lavish scale upon which numerical data are provided and by the all-round excellence of the work.

<div align="right">W. J.</div>

The Practice of Spectrum Analysis with Hilger Instruments, fifth edition, compiled by F. TWYMAN, F.R.S. Pp. 53. (London: Adam Hilger, Ltd.) 3s. 6d. net.

This booklet is largely a compilation of material contributed by several experienced workers on the spectrographic analysis of various substances, and is intended to serve as a guide to those taking up similar work. This purpose it will admirably fulfil. In the new edition each section of the fourth edition (1929) is brought up to date (August 1931), and new sections dealing with improved methods of quantitative analysis are added. There are three useful appendices: (1) on the determination of wave-lengths in a prism spectrogram by interpolation, (2) on the sensitization of photographic plates for the Schumann region by bathing, and (3) a bibliography of 82 papers and booklets selected from the large number that have appeared on the application of the spectrograph to chemical, metallurgical and mineralogical analyses.

<div align="right">W. J.</div>

Foundations and Methods of Chemical Analysis by the Emission Spectrum, being the authorised translation of *Die chemische Emissionsspektralanalyse*, by Dr WALTHER GERLACH and Dr EUGEN SCHWEITZER. Pp. 123. (London: Adam Hilger, Ltd., 1931.) 12s. 6d. net.

Many physicists will probably be surprised to read that the eminent investigator whose name they couple with one of the most fundamentally important experiments in present-day atomic physics (the Stern-Gerlach experiment), was, for the seven years ending mid-1929, engaged upon the problem of qualitative and quantitative chemical analysis by means of emission spectra. The valuable results of Prof. Gerlach's work in this field, partly in collaboration with Dr Schweitzer, are recorded in several papers in the *Zs. für anorg. und allg. Chemie* and other German chemical journals, and also form the nucleus of the very attractive book which, thanks to Adam Hilger, Ltd., now appears in our own language. The successful development of quantitative chemical analysis by means of emission spectra has brought many problems concerning metals within scientific control, and has provided new methods of investigation in connexion with many technical processes. The book deals with the development of the subject from its origin to the latest investigations, stress being laid upon the fundamental principles, practicability and trustworthiness of the method. It refers almost entirely to the use of spark spectra, describes many useful methods of procedure both with solid electrodes and with solutions, and deals with many particulars not completely treated in any other book. It is the most satisfying work on this subject that has come to the present reviewer's notice, and is well illustrated with 53 diagrams and reproductions of spectra.

<div align="right">W. J.</div>

Wave-length Tables for Spectrum Analysis, second edition, compiled by F. Twyman, F.Inst.P., F.R.S. and D. M. Smith, A.R.C.S., B.Sc., D.I.C. Pp. xi + 180. (London: Adam Hilger, Ltd.) 14s. 6d.

The words "spectrum analysis" as used in this title denote the spectrographic detection and estimation of the chemical elements, especially the metals, in mixtures, solutions, alloys, etc., and not the analysis and interpretation of atomic and molecular spectra—the subject to which Sir Arthur Schuster gave the name "spectroscopy." Since 1923, when Mr Twyman compiled the first edition of these very useful tables, much information has been obtained for the application of spectrographic methods to chemical and metallurgical analyses, and Mr Smith's present revision of the work is opportune. All the former tables of persistent lines in spark spectra have now been corrected to the I.Å. scale, and tables of persistent lines by Lundegårdh for flame spectra and by Ryde and Jenkins for arc spectra are now included, as well as the full list of ultimate lines by A. T. Williams, with series and temperature classifications and excitation potentials added. The tables are accompanied by explanatory notes and useful practical information both in the main text and also in an appendix. A brief theoretical account of the various types of spectrum, by Prof. Andrade, is transferred here from another Hilger publication.

The reviewer has very little to note in the nature of criticism. The French *raie* has been correctly translated as *line* everywhere except on p. 110 and pp. 115–18, where the unsatisfactory rendering *ray* appears. It is surprising that no mention is made of Kayser's very useful *Tabelle der Schwingungszahlen* (Hirzel, Leipzig, 1925) for the reduction of λ_{air} between 2000 and 10,000 Å. to ν_{vacuum}. No spectroscopist who has used it would, in spite of its many typographical errors, willingly return to the two-stage process mentioned in part I, namely reduction from λ_{air} to λ_{vacuum} by means of table 2 and subsequent conversion to ν_{vacuum} by means of a table of reciprocals.

W. J.

Photochemical Processes, A general discussion held by the Faraday Society. Pp. 216. (London: Faraday Society.) 10s. 6d.

As recently as 1925 photochemical processes were discussed by the Faraday Society. The fact that again in 1931 the same society should meet to consider the same subject was welcomed by Prof. Mecke, who opened the discussion, because of the growing interest in this branch of reaction chemistry. After the first meeting the views became general that for the primary photochemical reaction each absorbed quantum puts one and only one molecule into a reactive state; and that photochemical efficiencies not equal to unity must, therefore, be influenced by the secondary reactions that follow. These secondary reactions have received increasing attention since 1925.

Three out of the four lines of inquiry at this recent meeting were purely physico-chemical in scope. In part I were discussed, under the title "Molecular spectra in relation to photochemical change," the activation of the almost sluggish non-reacting molecules by the absorption of a light quantum $h\nu$. The reactions of the activated molecules in gaseous systems were considered in part II, and in solid and liquid systems in part III. Prof. Max Bodenstein introduced part II, and Prof. A. Berthoud led lucidly in part III. Of the great interest to physicists, chemists, and biologists who analyse photo-effects in living cells, of the new data presented and of the views put forward in these three parts there can be no doubt.

In part IV, under the title "Photosynthesis," is included a medley of papers. Two of these bear wholly on the title, in that in them are considered the synthesis of organic substances *in vitro* under the influence of light energy. The contribution by Prof. Baly, a well-known pioneer in this field, is sure to be widely read. Prof. Otto Warburg, a distinguished bio-chemist, gives very briefly his views concerning the energy-absorption

by green plant cells during the photo-reduction of carbon dioxide. He then deals with an interesting effect of light on the respiratory mechanism of cells. This is most certainly not photosynthetic in nature. The last paper in this part, on the "Measurement of ultra-violet radiation," seems to be misplaced under the title of the part. We do not know what aim those who planned the latter had in mind. Possibly it was intended to show the importance of the purely physico-chemical work in the analysis of the photo-biochemical reaction, taking place in living green plant cells, on which the food supplies of most living organisms ultimately depend. If this was the case, it is to be hoped that such a happy gesture to the green leaf will again be made when the society next discusses photochemical processes; but that then the primary activation of plant physiologists in this country will be followed by the secondary reaction of their taking part in the discussions.   M. J.

The Physics of High Pressure, by Prof. P. W. Bridgman, Ph.D. Pp. vii + 398. (London: G. Bell and Sons, Ltd.) 22s. 6d.

This is a monograph by a pre-eminent authority on a subject which he has made peculiarly his own. It deals fully with the technique of the production and measurement of high pressures, up to the order of 20,000 atmospheres, and with the remarkable variety of phenomena which can be observed in substances exposed to such pressures. The scope of the book may be gauged from this (incomplete) list: elasticity of solids at high pressures, with special sorts of rupture observed thereat; equations of state; melting phenomena; polymorphic transitions; effects of high pressure on electric, thermo-electric, optical and thermal properties and on viscosity; and even the effect of pressure in facilitating the germination of clover seeds! Many of these discussions are here collected together for the first time, and the resulting volume should be of absorbing interest to engineers and physical chemists, as well as to physicists. Certainly no one engaged in high-pressure work can afford to be without this volume, and no experimental physicist, whatever his main professional interest, can fail to be fascinated by it. The editor and publishers are to be congratulated upon securing the book for their series of "International Text-books of Exact Science."

H. R. R.

Die Elliptischen Funktionen von Jacobi, by L. M. Milne Thomson. Pp. xiv + 69. (Berlin: Julius Springer.) Geb. 10.50 marks.

The functions sn u, cn u, dn u are tabulated separately for values of the argument at intervals of 0·01, and for values of k^2 at intervals of 0·1; in every case, except of course for $k^2 = 1$, the ranges covered exceed a quarter-period. The first differences are conveniently inserted between the consecutive entries. These tables, as is explained in the preface, are entirely new, and for many purposes they are much more convenient than those hitherto available, which tabulate u against the amplitude ϕ. They are followed by eight-figure tables of the complete integrals K, K', E, E', and of q and q_1, for values of k^2 at intervals of 0·01. It would be useful if a table of Γ (a) could be added in any future re-issue. There is a short introduction with a convenient collection of formulae. The arrangement and printing are admirably clear, and this handy little volume will be a valuable addition to the reference library.

G. S. L.

Numerical Examples in Physics, by W. N. Bond. Pp. 128. (London: E. Arnold and Co.) 4s.

There is little glory and much toil in the writing of text-books; and this is peculiarly true of the compiling of books of examples. All the more seemly is it to thank Dr Bond

for facing the ungrateful task and for providing us with a really well chosen and comprehensive selection of some 450 examples. A few of these are of intermediate standard; the great majority will give good practice to pass and to honours students. Not all the questions are numerical—a number provide subjects for brief essays. It would be very helpful, especially to the student working unaided, if the book contained an appendix giving some sources of information on the subject-matter of these essays, and we hope that Dr Bond will supply this information in the second edition which will, we trust, be necessary.

The book may be unreservedly commended.   A. F.

Problems in Physics, second edition, by WILLIAM D. HENDERSON, Ph.D. Pp. ix + 245. (London: McGraw Hill Publishing Co., Ltd.) 11*s*. 3*d*.

This book contains a collection of 825 problems in mechanics and physics originally designed to accompany the courses in physics arranged for students of engineering in the University of Michigan. About two-thirds of the problems deal with mechanics and electricity; the remainder with surface tension, diffusion, heat, light, sound and radiation. Each section is prefaced by a short statement of the fundamental principles and usually also by one or two worked examples. The standard corresponds approximately to that of the intermediate classes of the English universities; exceptionally there are sections on technical electricity (e.g., a.c. phenomena and electric generators) which would not fall within intermediate physics syllabuses in this country. The fact that a second edition has been called for shows that there is a demand for the book in the United States. English text-books usually contain an adequate collection of numerical problems and the reviewer imagines that few teachers over here would require their students to purchase a book costing 11*s*. for their exercise work. The problems themselves are straightforward and practical rather than ingenious. The text is good and the author neither omits nor muddles his units. The only bad error appears on p. 138 where Dr Henderson has permitted himself to write "The electromotive force (e.m.f.) of an electric generator (battery or dynamo) is its capacity for generating electric pressure or potential...e.m.f. is measured in terms of work, ergs." The book closes with 8 pages of data and an index. It can be recommended for the use of teachers too busy to devise problems themselves.

D. O. W.

Electricity and Magnetism, by VINCENT C. POOR, Ph.D. Pp. ix + 183. (London: Chapman and Hall.) 11*s*.

This work, by the Associate Professor of Mathematics, University of Michigan, belongs to the class of text-books of mathematical physics. The treatment is based mainly on the methods of vector analysis, and the book is put forward not as a comprehensive treatise but rather as "a path through" the subject of electrical theory. The author also expresses the view that a course such as this should be taken not only by the student of physics but also by the good electrical engineer before he leaves college.

After a concise introduction to vector analysis, theorems being stated but not proved, come four chapters treating of the classical side of electricity and magnetism: one on electrostatics, one on magnetism and dielectric polarization, one on current electricity, and one on the dynamics of the electric current. In the last-mentioned chapter the author expounds d'Alembert's principle, Hamilton's principle and Lagrange's equations, and treats the electric current as a cyclic mechanical system. The final chapter v is in two parts. The first, entitled "Electron theory," includes a treatment of the electromagnetic field from the Lorentz point of view, a derivation of ether stresses in matrix form, and a brief treatment of radiation pressure. The second part of chapter v is entitled "Special

relativity and the electrodynamic theory," and in it the invariance of Maxwell's equations is shown and the expressions for the longitudinal and the transverse mass of the Lorentz electron are derived. Exercises are set at the end of the earlier chapters. The book is not exactly easy reading, but should prove of much value to the advanced student under the guidance of a competent teacher.

<div align="right">D. O.</div>

Magnetic, Meteorological and Seismographic Observations made at the Government Laboratories, Bombay and Alibag, in the year 1927, under the direction of S. K. BANERJI, D.Sc. Pp. iii + 132. (Calcutta: Indian Government.) 17s. 6d.

This volume is the 49th issue and brings the record of meteorological and magnetic observations up to 82 years. The mean temperature during the year was 79·9° F., 0·3° F. below the normal, and the rainfall 74·4 in., 3·8 in. above the normal. The horizontal magnetic field at Alibag was 0·371 and slowly increased; deviation of the compass 1·9° west, an increase of 2·7° west on 1926. The dip was 25° 26', an increase of 3'. There were 5 days of very great and 8 of great magnetic disturbance, and one seismic disturbance of considerable amplitude on December 28 at 19 h. 2 m. 11 s. Greenwich mean time.

<div align="right">C. H. L.</div>

The Observatories Year Book, 1929 (M.O. 330), published by the authority of the Meteorological Committee. Pp. 441. (London: H.M. Stationery Office.) £3. 3s. 0d.

This is the eighth issue of the Year Book. It contains daily records of the meteorological elements at the principal first-order stations—Aberdeen, Eskdalemuir, Valencia and Kew—of atmospheric electricity and terrestrial magnetism at Lerwick and Eskdalemuir, and of atmospheric electricity, atmospheric pollution, seismology and aerological data obtained with sounding-balloons at Kew. Each station reports that 1929 was a normal year.

<div align="right">C. H. L.</div>

Meteorological Office 331c. *Geophysical Memoirs No.* 53. *Characteristics of Rainfall Distribution in Homogeneous Air Currents and at Surfaces of Discontinuity*, by A. H. R. GOLDIE, M.A., F.R.S.E. Pp. 19. (London: H.M. Stationery Office.) 1s. 3d.

The author shows that a depression does not produce rain independently of the time of day or of the region over which it is passing. The time-effect appears greater in warm or cold fronts than in homogeneous equatorial or polar currents. In warm fronts it seems to depend on the isolation and radiation of the cloudy warm air ascending at the front, and in cold fronts on that of the warm air ahead of the front.

<div align="right">C. H. L.</div>

Journal of Research of the Bradford Technical College, Vol. 1 (1930). Pp. xii + 295. (Bradford Education Committee.)

In this volume are collected the records of thirty-three researches, the work of students and members of the staff of Bradford Technical College. From an institution whose work must be closely allied to that of a surrounding industrial area one would expect a preponderance of technical researches: yet although the majority of papers in this collection are of that kind, subjects of "pure" scientific interest are to be found in several notable

contributions. An investigation by T. W. Price of the decomposition of carbamyl chlorides substituted by hydroxyl compounds and a general solution of $\nabla^2 \psi = w$ in n-dimensional Euclidean space by A. J. Carr should satisfy the most unsullied of purists. The subjects, indeed, range widely from matters of purely industrial interest, such as the properties and peculiarities of textile fibres and the design of machine components, to the detailed analysis of power-transmission by belts; from the analysis by chemical means of the constituents of "union materials" in which different types of fibre are inextricably mingled, to the measurement of dielectric losses in coils; from the cataloguing of algae in a bog near Bradford to the measurement of the optical properties of gaseous carbon disulphide.

Among so varied a collection considerable differences of quality are to be expected: nevertheless most of the papers have already been printed in recognized scientific and industrial journals, a guarantee at least of usefulness. In point of fact, several papers are not only of considerable scientific value, but excellently presented. They are here beautifully reprinted, and form a collection on which the contributors may be congratulated and which could occupy a useful place in the collection of any investigator.

J. P. A.

Proceedings of the Institution of Mechanical Engineers, Vol. 11, June to December, 1930. Pp. viii + 614.
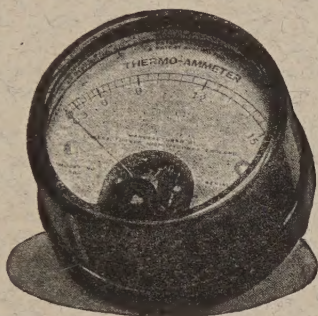
This volume contains two or three papers which should be of considerable interest to physicists; of these perhaps the most important is one on "The coefficients of heat-transfer from tube to water," which is a report of an investigation carried out by Messrs Eagle and Ferguson under the direction of a committee of prominent engineers, experienced in condenser design, formed by the British Electrical and Allied Industries Research Association. The committee have looked into some 200 papers on the problem of heat-transfer and have superintended a very admirable and thorough piece of research. The paper is of the nature of an interim report; the work done so far only deals with the transfer of heat from a hot tube to water circulating through it, and the work will be followed by an investigation into the transmission of heat from the tube to the medium surrounding it. This promises to be the most thorough research of its kind which has yet been done on the problem of heat-transfer. Mr V. E. Pullin, Director of Radiological Research, Woolwich, writes an interesting paper on the subject of "X-rays in engineering practice." The paper is largely confined to radiography in its application to the examination of castings and forgings for flaws. One realizes the remarkable development which has taken place in the technique of this branch of science when one considers that by applying a voltage of 240,000 to an X-ray tube it is possible to penetrate a steel forging 4 in. thick. Prof. J. J. Guest contributes a paper on "The effects of rapidly acting stress." In his analysis of his results he concludes "that the stress which is important in design... is very little affected by the suddenness or the brief duration of the loading," and his work would suggest that there is room for more research on the problem. Dr Gough in his paper on "The effect of low temperature on the shock-resisting properties of new wrought-iron chain" concludes that the shock-absorbing value of a chain at low temperatures depends largely on the welding of the links, as one might expect; best-quality chain iron does not develop any brittleness in the absence of notches, even at temperatures as low as $-78°$ C.

Mr L. St L. Pendred in his presidential address makes some important "random reflections." He deplores the necessity for extreme specialization in engineering, and appeals to mechanical engineers to adventure more into the domain of physics.

The 17th Thomas Hawksley Lecture was this year delivered by Prof. J. W. Gregory, F.R.S., on "The machinery of the earth."

G. A. W.

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